

78465

**M. Sc. Mathematics 4th Semester
(CBCS Scheme) Examination – May, 2024**

GRAPH THEORY

Paper : 17MAT24DA2

Time : Three hours]

[Maximum Marks : 80

Before answering the questions, candidates should ensure that they have been supplied the correct and complete question paper. No complaint in this regard, will be entertained after examination.

Note : Attempt five questions in all, selecting one question from each Section. Question No. 1 (Section-I) is compulsory. All questions carry equal marks.

SECTION – I

- 1. (a) Give one example of a graph : 12**
- (i) Having no odd vertices.**
 - (ii) Having no even vertices.**
 - (iii) Having exactly one odd vertex.**
 - (iv) Having exactly one even vertex.**
 - (v) Having exactly two odd vertices.**
 - (vi) Having exactly two even vertices.**

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P. T. O.

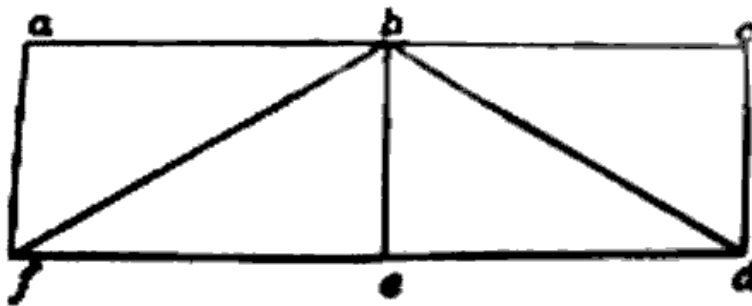
(b) Give an example of a disconnected graph with four components where each component is complete. 4

2. (a) Prove that a non-trivial simple graph G must have at least one pair of vertices whose degrees are equal. 8

(b) If there is a path from v_1 to v_2 in a graph with n vertices then there does not exist a path more of than $(n - 1)$ edges from vertex v_1 to v_2 . 8

SECTION - II

3. (a) Does the graph

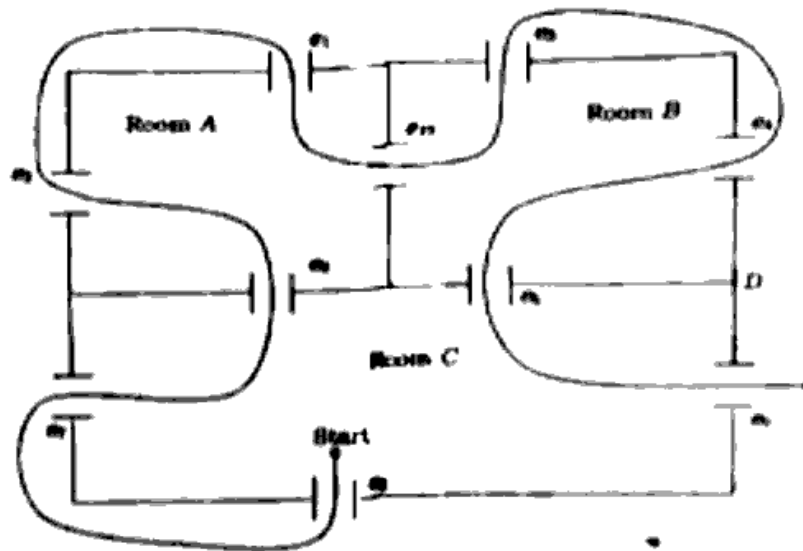


has a Hamiltonian circuit. 8

(b) Show that a connected planar graph with 7 vertices and degree of each vertex equal to 4 must have 8 regions of degree 3 and 1 region of degree 4. 8

4. (a) If G is a connected graph and every vertex of G has even degree then G has an Euler circuit. 8

- (b) In the below structure of rooms, can a person walk through all rooms crossing every door exactly once. 8



SECTION - III

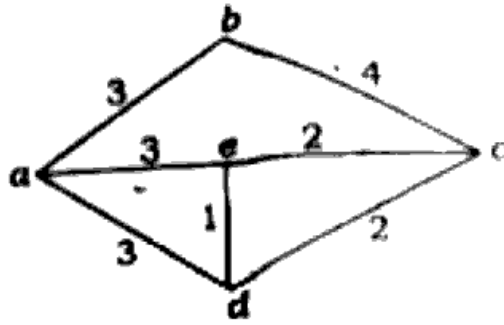
5. (a) The degrees of the vertices of a certain tree T of order 13 are 1, 2 and 5. If T has exactly three vertices of degree 2, how many end vertices does it have? 8
- (b) Prove that a tree with n vertices has $n - 1$ edges. 8
6. (a) Find the minimum number of extension codes, each having four outlets require to connect 22 bulbs to a single electric outlet. 8
- (b) Prove that a tree that has more than one vertex has at least one vertex of degree one. 8

SECTION - IV

7. (a) Build the minimal spanning tree of the following graph using Prim's algorithm. 8

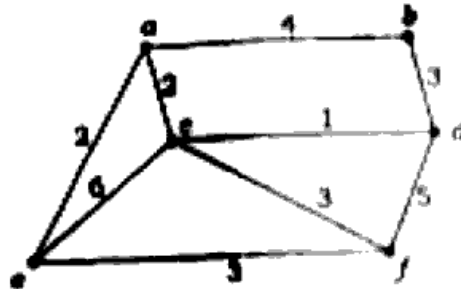
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P. T. O.



(b) Find all the spanning trees of K_4 . 8

8. (a) Using Kruskal's algorithm, find the minimal spanning tree for the graph given below : 8



(b) Let G be a connected graph with n -vertices then G is a tree iff every edge of G is bridge (cut-edge) and so, we can say that every edge in a tree is a bridge. 8

SECTION - V

9. (a) Define walks and paths. 2 × 8 = 16

(b) Define connected graphs.

(c) Define Hamiltonian paths.

(d) State Kuratowski theorem.

(e) Discuss prefix code.

(f) Discuss tree traversal.

(g) Define Cut Sets.

(h) Define minimal spanning tree.