25063

M.Sc. 3rd Semester (DDE) Examination,

December–2022

MATHEMATICS

Paper–21MAT23C3

Fluid Dynamics

Time allowed : 3 hours] *[Maximum marks : 80*

Note : *Attempt one question from each of the* ***Sections I-IV. Section–V is compulsory.*** *All questions carry equal marks.*

Section–I

1. (a) Obtain the streamlines, pathlines and streaklines passing through $(\ell, \ell, 0)$ at $t = 0$ for the flow $u = \frac{x}{t_0}\left(1 + \frac{t}{t_0}\right), v = \frac{y}{t_0}, w = 0$ where ℓ and t_0 are constants having the dimensions of length and time respectively.

   (b) If σ (s) is the cross-sectional area of a stream filament, prove that the equation of continuity is $\frac{\partial}{\partial t}(\rho \sigma) + \frac{\partial}{\partial s}(\rho \sigma q) = 0$, where δs is an element of arc of the filament and q is the fluid speed.

2. (a) Find the restriction of f_1, f_2, f_3 such that $\frac{x^2}{a^2} f_1(t) + \frac{y^2}{b^2} f_2(t) + \frac{z^2}{c^2} f_3(t) = 1$ is a possible form of boundary surface of a moving fluid.

   (b) Obtain the components of acceleration at a point of fluid in cylindrical co-ordinates.

Section–II

3. (a) Show that the pressure at a point of a moving fluid is same in all direction.

   (b) Derive the Bernoulli's equation for unsteady flow.

4. (a) Show that the rate of change of total energy of any region of the fluid, as it moves under conservative body force, is equal to the rate of work done by the pressure over the boundary surface.

   (b) State and prove Kelvin Circulation Theorem.

Section–III

5. Find the velocity potential of the problem of accelerating sphere moving in a fluid at rest at infinity. Also find the kinetic energy of the system, pressure at a point on the sphere and equation of motion of the sphere.

6. (a) Obtain the expression for kinetic energy generated by impulsive motion.

   (b) Describe the image of a doublet in a solid sphere.

Section–IV

7. (a) Explain the physical interpretation of the Lagrange's stream function.

   (b) A source and a sink of equal strength are placed at the points $\left(\pm\frac{a}{2}, 0\right)$ within a fixed circular cylinder $x^2 + y^2 = a^2$. Show that the streamlines are given by

$$\left(r^2 - \frac{a^2}{4}\right)(r^2 - 4a^2) - 4a^2y^2 = ky\,(r^2 - a^2),$$

where k is a constant.

8. Between the fixed boundaries $\theta = \pi/6$ and $\theta = -\pi/6$, there is a two-dimensional liquid motion due to a source at the point $(r = c, \theta = \alpha)$ and a sink at the origin, absorbing water at the same rate as the source produces it. Find the stream function and show that one of the streamlines is a part of the curve

$$r^3 \sin 3\alpha = c^3 \sin 3\theta.$$

Section–V

9. (i) What do you mean by two-dimensional motion ?

   (ii) State Blasius theorem.

   (iii) For the given stream function $\psi = x^3 - 3xy^2$, check whether the flow is rotational or irrotational.

   (iv) State kelvin minimum energy theorem.

   (v) Write short note on impulsive motion.

   (vi) Define flux across a surface.

   (vii) Differentiate between path lines and streaklines.

   (viii) Introduce the concept of complex potential.