# M.Sc. 3rd Semester (DDE) Examination,

## December–2022

## MATHEMATICS

## Paper–21MAT23C2

## Elementary Topology

*Time allowed : 3 hours ]* *[ Maximum marks : 80*

***Note :*** *Attempt **five** questions in all, selecting **one** from each section **I-IV**. **Section–V is compulsory.** All questions carry equal marks.*

### Section–I

1. (a) Let T denote usual topology and U denote upper limit topology for the set R. Show that the topologies T and U are comparable. 8

   (b) For any set A in a topological space (X, T), show that $A^0 = [C(A^C)]^C$, where $C(A^C)$ is closure of $A^C$. 8

2. (a) Show that A set is open iff it is neighbourhood of each of its points. 8





   (b) Let (X, T) be a topological space and b (A), $\bar{A}$ and $A^0$ be the boundary, closure and interior of A respectively. Then prove that

   (i) A is closed iff $b(A) \subseteq A$

   (ii) A is open iff $A \cap b(A) = \phi$ 8

### Section–II

3. (a) Let (X, T) be a topological space and A be a subset of X. Let $\mathcal{B}$ be a base for T. Show that $\mathcal{B}_A = \{B \cap A ; B \in \mathcal{B}\}$ is a base for the sub-base topology on A. 8

   (b) State and prove the pasting lemma about continuous functions. 8

4. (a) If every two points of a set E are contained in some connected subset C of E then show that E is a connected set. 8

   (b) Prove that a space X is locally connected if and only if for every open set U of X, each component of U is open in X. 8

## Section–III

5. (a) Let $f : X \to Y$ be a continuous mapping of X into Y and E be a compact subset of X then prove that f (E) is compact in Y. 8

(b) Let (X, T) be a $T_1$-space. Then show that X is countably compact if and only if it has the Bolzano Weierstrass property. 8

6. (a) Define locally compact space. Show that every compact topological space is locally compact. Give an example to show that converse is not true. 8

(b) Define sequentially compact space and give an example of a compact space that is not sequentially compact. 8

## Section–IV

7. (a) Show that an infinite Hausdorff space X contains an infinite sequence of non-empty disjoint open sets. 8

(b) Prove that the property of a space being separable is a topological property. 8





8. (a) If f is a mapping of the first axiom space (X, T) into the topological space (X*, T*), then show that f is continuous at $x \in X$ iff for every sequence $<x_n>$ of points in X converging to $x \in X$, the sequence $<f(x_n)>$ converges to f (x). 8

(b) Prove that closed subspace of a Lindelof space is lindelof. 8

## Section–V

9. (a) Define upper limit topology on the set of real numbers. How does it compare with the usual topology ?

(b) Define closure operator and give an example to show that $C(A \cap B) \neq C(A) \cap C(B)$.

(c) Let (X*, T*) be the subspace of (X, T) then every open subset of X* is open in X iff X* is open in X.

(d) Give an example of a topological space which is not connected.

(e) Is completeness topological property ? Justify.

(f) Define finite intersection property.

(g) A topological space may not be $T_0$ space if a topology coarser than it is $T_0$. Comment.

(h) Give an example to show that $T_1$ space is not $T_2$ space. $8 \times 2 = 16$