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# 74463

## M. Sc. Mathematics 2nd Semester
## CBCS Scheme w.e.f. 2016-17
## Examination – July, 2021
### INTEGRAL EQUATIONS AND CALCULUS OF VARIATIONS

**Paper : 16MAT22C3**

*Time : Three Hours ]* *[ Maximum Marks : 80*

*Before answering the questions, candidates should ensure that they have been supplied the correct and complete question paper. No complaint in this regard, will be entertained after examination.*

***Note :*** Attempt ***five*** questions in all, selecting ***one*** question from each Section. Question No. 9 (Section-V) is ***compulsory***. All questions carry equal marks.

### SECTION – I

1. (a) Describe the solution of Volterra integral equation by the method of successive substitution. 8

(b) Solve the following Volterra equation of the first kind after reducing it to a Volterra equation of the second kind : $\sin x = \int_0^x e^{x-1} u(t)\,dt$. 8



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2. (a) From an integral equation for the initial value problem $\frac{d^2y}{dx^2} - \sin x \frac{dy}{dx} + e^x y = x,\ y(0) = 1,\ y'(0) = -1$. 8

(b) Find the resolvent kernel to solve the integral equation $u(x) = 1 + x^2 + \int_0^x \frac{1+x^2}{1+\xi^2} u(\xi)\,d\xi$. 8

### SECTION – II

3. (a) Find the resolvent kernel for the Fredholm integral equation $u(x) = 1 + \lambda \int_0^1 (1 - 3xt) u(t)\,dt$, using the Neumann series expansion and hence solve the integral equation. 8

(b) Obtain the interated kernels for the following kernel $k(x,t) = e^x \cos t,\ a = 0,\ b = \pi$. 8

4. (a) Use the method for separable kernel to solve $u(x) = x + \int_0^1 (1 + x\xi) u(\xi)\,d\xi$. 8

(b) Transform the BVP $u''(x) = \lambda u(x)$ in $a < x < b$, $u(a) = 0,\ u(b) = 0$ to an equivalent Fredholm integral equation. 8

### SECTION – III

5. (a) Discuss the method for constructing the Green's function for a non homogeneous second order BVP by using the method of variation of parameters. 8

(b) Construct the Green's function for the BVP $u''(x) = f(x)$ in $a < x < 1$, $u(0) = 0, u(1) = 0$ and solve the BVP for $f(x) = e^x$. 8

6. (a) Reduce the B.V.P. $\frac{d^2u}{dx^2} + \lambda u = x$, in $0 < x < \frac{\pi}{2} u(0) = 0$, $u\left(\frac{\pi}{2}\right) = 0$ to a Fredholm integral equation, using the Green's function method. 8

(b) Use the Hilbert Schmidt theory for symmetric kernels to solve the Fredholm equation. $u(x) = x + \lambda \int_0^1 K(x,t)u(t)\,dt$ with $K(x,t) = \begin{cases} x(1-t), x \le t \\ t(1-x), t \le x \end{cases}$. 8

## SECTION – IV

7. (a) Derive the Euler's equation for the functional $J[y] = \int_a^b f(x,y,y')dx$, $y(a) = A$, $y(b) = B$ to have an extremum. 8

(b) Among all curves of length $l$ in the upper half-plane passing through the points $(-a, 0)$ and $(a, 0)$, find the one which together with the interval $(-a, a)$ encloses the largest area. 8

8. (a) What do you mean by a functional and its differential ? Show that the differential of a differentiable functional is unique. 8





(b) Find the external of the functional $J[y,z] = \int_0^{\frac{\pi}{2}} [(y')^2 + (z')^2 + 2yz]dx$, $y(0) = 0, y\left(\frac{\pi}{2}\right) = 1$, $z(0) = 0, z\left(\frac{\pi}{2}\right) = -1$. 8

## SECTION – V

9. (a) Define variation of a functional.

(b) Define weak extremum.

(c) Show that the function $y(x) = (1+x^2)^{-\frac{3}{2}}$ is solution of the Volterra integral equation $y(x) = \frac{1}{1+x^2} - \int_0^x \frac{\xi}{1+x^2} y(\xi)d\xi$.

(d) What do you mean by homogeneous integral equation ?

(e) Define resolvent kernel for Fredholm integral equation.

(f) Define separable kernel and given an example.

(g) Hilbert Schmidt theorem (statements only).

(h) Define a self-adjoint operator. 2 × 8 = 16