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M. Sc. (Math with Computer Science) 2nd Semester CBCS Scheme w.e.f. 2016-17 Examination – June, 2023

INTEGRAL EQUATIONS AND CALCULUS OF VARIATIONS

Paper : 16MMC22C3

Time : Three Hours] *[Maximum Marks : 80*

Before answering the questions, candidates should ensure that they have been supplied the correct and complete question paper. No complaint in this regard, will be entertained after examination.

Note : Attempt ***five*** questions in all, selecting ***one*** question from each Section. Question No. 9 **(Section – V)** is ***compulsory***. All questions carry equal marks.

SECTION – I

1. (a) Reduce the IVP $y'' + \mu y = 0,\ y(0) = 1,\ y'(0) = 0$ into integral equation.

   (b) Solve $y(x) = 2 + \int_0^x (x - 3\xi)\, u(\xi)\, d\xi$ using successive approximation.

2. (a) Solve the integral equation :

$$\sin x = \lambda \int_0^x e^{2(x-t)} y(t)\,dt$$

(b) Discuss the method of iterated kernels to solve volterra integral equation.

SECTION – II

3. (a) Reduce the BVP $y'' - 2y = 0,\ a < x < b\ y(a) = 0 = y(b)$ into Fredholm integral equation.

(b) Explain the method of successive approximation to solve Fredholm integral equation.

4. (a) Solve the integral equation and discuss all its possible cases with the method of separable Kernel $y(x) = f(x) + \lambda \int_0^1 (1 - 3xt) y(t)\,dt$.

(b) Prove that the eigen functions corresponding to two different eigen values are orthogonal over (a, b) for the Fredholm integral equation $y(x) = \lambda \int_a^b K(x,t) y(t)\,dt$ with symmetric Kernel.

SECTION – III

5. (a) Construct the Green's function for the BVP $\frac{d^2u}{dx^2} + \mu^2 u = 0$ with the condition $u(0) = u(1) = 0$.

(b) Reduce the BVP $\frac{d^2u}{dx^2} + xu = 1,\ u(0) = u(1) = 0$ to an integral equation.

6. Discuss the construction of Green's function by variation of parameter method.

SECTION – IV

7. (a) Find the shortest distance between two points in a plane is a straight line.

(b) Derive the necessary condition for the extremum of a functional dependent on higher derivatives.

8. (a) Find the extremals of the functional

$$\int_{\theta_1}^{\theta_2} \sqrt{r^2 + r'^2}\, d\theta \text{ where } r = r(\theta)$$

(b) Find the extremals of the functional

$$J[y, z] = \int_{x_0}^{x_1} (2yz - 2y^2 + y'^2 - z'^2)\, dx$$

SECTION – V

9. (a) Define Volterra integral equation of second kind.

(b) What do you mean by difference Kernel ?

(c) Define resolvent Kernel.

(d) What do you mean by orthogonality of two functions ?

(e) Define self-adjoint operator.

(f) If $\alpha(x)$ is continuous in $[a, b]$ and if $\int_a^b \alpha(x) h(x)\, dx \quad 0$ for every function $h(x) \in C[a, b]$ such that $h(a) = h(b) = 0$, then $\alpha(x) = 0$ for all $x \in [a, b]$.

(g) Define Green's function.

(h) Find the external of the functional $J[y] = \int_a^b (x - y)^2\, dx$.