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M. Sc. Math 2nd Semester CBCS Scheme

w.e.f. 2016-17 Examination,

July-2022

INTEGRAL EQUATIONS AND CALCULUS OF VARIATIONS

Paper-16MAT22C3

Time allowed : 3 hours] *[Maximum marks : 80*

Note : *Attempt **five** questions in all. Questions No. 1 is **compulsory**.*

Section-I

1. (a) From an integral equation for the initial value problem : 8

$$\frac{d^2y}{dx^2} - 2x\frac{dy}{dx} - 3y = 0,\ y(0) = 1,\ y'(0) = 0$$

(b) Using successive approximation method, solve

$$u(x) = 1 - \int_0^x (x-t)\,u(t)\,dt,\ \text{taking } u_0(x) = 1. \quad 8$$

2. (a) Solve the integral equation by using Laplace transform $u(x) = 1 + \int_0^x \sin(x-t)\,u(t)\,dt$. Also verify your result. 8

(b) Find the resolvent Kernel to solve the Volterra integral equation $u(x) = f(x) + \lambda\int_0^x e^{x-\xi}u(\xi)\,d\xi$. 8

Section-II

3. (a) Describe the method of successive approximations to solve Fredholm integral equation 8

(b) Solve the Fredholm integral equation

$u(x) = \cos x + \lambda\int_0^{\pi} \sin(x-t)\,u(t)\,dt$ by the method of separable kernel. 8

4. (a) Find the resolvent kernel for the Fredholm integral equation https://www.mdustudy.com

$u(x) = 1 + \lambda\int_0^1 (1-3xt)\,u(t)\,dt$ using Neumann series expansion. 8

(b) Solve the integral equation

$u(x) = 1 + \lambda\int_0^1 (1-3x\xi)\,u(\xi)\,d\xi$ by the method of iteration. Also find the condition on λ for which solution exists. 8

Section-III

5. (a) Construct the Green's function for the BVP $u''(x) + u(x) = f(x)$ in $0 < x < 1$, $u(0)=0, u(1)=1$ by using its properties. 8

(b) Reduce the BVP $u''(x) + \lambda u(x) = x,\ 0 < x < \frac{\pi}{2}$

$u(0) = 0,\ u\left(\frac{\pi}{2}\right) = 0$ to a Fredholm integral equation, using the Green's function method. 8

6. Define the Green's function of BVP :

$$\frac{d}{dx}\left[r(x)\frac{du}{dx}\right] + \left[q(x) + \lambda\rho(x)\right]u(x) = f(x) \text{ in } a < x < b,$$

$$\alpha_1 u(a) + \alpha_2 u'(a) = 0$$

$$\beta_1 u(b) + \beta_2 u'(b) = 0$$

and construct $G(x,\xi)$ by using the basic four properties. 16

Section-IV

7. (a) Define the variation of a functional. Show that a necessary condition for a differentiable functional to have an extremum for $y = \hat{y}$ is that its variation vanishes for $y = \hat{y}$. 8

(b) Among all the curves joining two given points (x_0, y_0) and (x_1, y_1), find the one which generates the surface of minimum area when rotated about the x-axis. 8

8. (a) Solve the variational problem

$$J[y] = \int_1^2 \left\{ \frac{\sqrt{x^2 + (y')^2}}{x} \right\} dx,\ y(1) = 0,\ y(2) = 1$$ 8

(b) Find the geodesics of the circular cylinder $\vec{r} = (a\cos\phi,\ a\sin\phi,\ z)$ 8

Section-V

9. (a) Define Volterra integral equation of second kind and give an example :

(b) Define Leibnitz's rule for differentiation under integral sign.

(c) What do you mean by non-homogenous integral equation of Fredholm type ?

(d) What is Fredholm integral equation of first kind and give an example ?

(e) State Hilbert-Schmidt theorem for symmetric kernels.

(f) Define a self-adjoint operator.

(g) What is Brachistochrone problem ? Give an example.

(h) What is problem of Geodesics ? Give an