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86094

Master of Science Mathematics 1st Sem. Examination – December, 2024

MATHEMATICAL STATISTICS

Paper : 24MAT201DS04

Time : Three Hours] *[Maximum Marks : 70*

Before answering the questions, candidates should ensure that they have been supplied the correct and complete question paper. No complaint in this regard, will be entertained after examination.

Note : Attempt ***five*** questions in all, selecting ***one*** question from each Section. Question No. 1 is ***compulsory***. All questions carry equal marks.

1. (a) Give statistical definition of probability. 1½

(b) What are 'pairwise' and 'mutually' independent events ? 2

(c) Define a random variable. Give an example. 2

(d) For a random variable X, show that

$$V(aX + b) = a^2 V(X),$$

where 'a' and 'b' are constants. 1½

(e) Find moment generating function of a geometric distribution. 2

(f) State under what conditions binomial and Poisson distributions tends to normal distribution. 1½

(g) Explain the term 'critical region' in testing of hypothesis. 1½

(h) Differentiate between 'Simple' and 'Composite' hypotheses. 2

SECTION – I

2. (a) What is axianatic definition of probability ? For two events A and B prove that for $B \subseteq A$

$$P(B) \leq P(A).$$ 4

(b) State and prove multiplication law of probability. 6

(c) A bag contains 6 white and 9 black balls. This drawings of 4 balls each are made from the bag such that the balls are not replaced before the second trial. What is the probability that the first drawing will give 4 black balls and second 4 white balls ? 4

3. (a) Define conditional probability. Let A, B and C be any three events such that B and C are independent, show that 6

$$P(A/B) = P(A/B \cap C) \, . \, P(C) + P(A/B \cap \overline{C}) \, . \, P(\overline{C})$$

(b) The odds that a book on statistics will be favourably reviewed by 3 independent critics are 3 to 2, 4 to 3 and 2 to 3 respectively. Then, obtain the probability that of three reviews atleast one of the review will be fovourable. 5

(c) What is the Baye's theorem in probability ? Give its uses. 3

SECTION – II

4. (a) Explain the following terms : 6
   (i) Probability density function
   (ii) Joint probability mass function
   (iii) Marginal probability density function, and
   (iv) Conditional distribution function

   (b) A two-dimensional random variable (X, Y) has a joint p.m.f. :

$$p(x, y) = \frac{1}{27}(x + 2y),$$

   for x and y can assume only the integer values 0, 1, and 2.
   Find (i) Marginal distribution of X and Y, and (ii) Conditional distribution of Y for $X = x$. 8

5. (a) What do you mean by mathematical expectation of a random variable ? For two discrete random variables, prove that

$$E(X + Y) = E(X) + E(Y).$$ 6

   (b) Let X be a random variable taking values 1, 2, 3 and 4 with probability 0.1, 0.2, 0.3 and 0.4, respectively. Find $V(2X + 3)$. 4

   (c) Write a note on moment generating function and its properties. 4

SECTION – III

6. (a) Define the binomial distribution. Obtain its mean and variance. Show that mean ≥ variance. 8

   (b) State and prove 'lack of memory property' of the exponential distribution. 6

7. (a) If X is uniformly distributed with mean 1 and variance 4/3 then find $P(X > 0)$ and $P(X < 2)$. 5

   (b) What do you mean by a normal distribution ? State chief characteristics of the normal distribution and the normal curve. 9

SECTION – IV

8. (a) Distinguish between the following : 9
   (i) Parameter and statistics
   (ii) Null and alternative hypotheses
   (iii) One-tailed and two-tailed tests.

   (b) What is standard error of an estimate ? Explain, giving a suitable example. Also, give its utility. 5

9. (a) Explain test of significance of 'difference of two proportion' for large samples. 5

   (b) In a random sample of 500, the mean is found to be 20. In another independent sample of 400, the mean is 15. Test whether the samples have been drawn from the same population with standard deviation 4. [Given that $z_{.05} = 1.96$]. 6

   (c) In testing of hypothesis, what are two kinds of errors ? Explain. 3