Roll No. ....

86093

Master of Science Mathematics 1st Semester Examination – December, 2024

COMPLEX ANALYSIS

Paper : 24MAT201DS03

Time : Three hours] *[Maximum Marks : 70*

Before answering the questions, candidates should ensure that they have been supplied the correct and complete question paper. No complaint in this regard, will be entertained after examination.

Note : Attempt ***five*** questions in all, selecting ***one*** question from each Section. Question No. **9** (Section-**V**) is ***compulsory***. All questions carry equal marks.

SECTION – I

1. (a) For the function $f(z)=\begin{cases}(\bar{z})^2/z, & z\neq 0\\ 0, & z=0\end{cases}$, $z=x+iy$, show that $f(z)$ is not differentiable at the origin, although C-R equations are satisfied at that point. 7

(b) State and prove sufficient condition for $f(z)$ to be analytic. 7

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2. (a) Prove that $u(x, y) = e^{-x}(x \sin y - y \cos y)$ is harmonic and find the conjugate harmonic function $v(x, y)$ such that $f(z) = u + iv$ is analytic. 7

(b) Discuss the branches, branch cut and branch point for the function $w = z^{1/2}$. 7

SECTION – II

3. (a) If $f(z)$ is analytic in a ring shaped region bounded by two closed contours C_1 and C_2 and Z_0 is a point in the region between C_1 and C_2, then show that $f(z_0)=\frac{1}{2\pi i}\int_{C_2}\frac{f(z)}{z-z_0}dz-\frac{1}{2\pi i}\int_{C_1}\frac{f(z)}{z-z_0}dz$, where C_2 is the outer contour. 7

(b) State and prove Poisson's integral formula. 7

4. (a) Show that the bounded entire functions are the constant functions. 6

(b) State and prove Taylor's theorem. 8

SECTION – III

5. (a) Expand $f(z)=\frac{1}{z(z-3)}$ in a Laurent's series valid for $1 < |z+1| < 4$. 7

(b) If an analytic function $f(z)$ has a pole of order m at $z = a$, then show that $\frac{1}{f(z)}$ has a zero of order m at $z = a$ and conversely. 7

6. (a) Let $f(z)$ be analytic in a domain D defined by $|z| < R$ and let $|f(z)| \le M$ for all z in D and $f(0) = 0$, then $|f(z)| \le \frac{M}{R}|z|$. Also, if the equality holds for any one z, then $f(z) = \frac{M}{R} z e^{i\alpha}$ where α is real constant. 7

(b) State and prove Inverse Function theorem. 7

SECTION – IV

7. (a) Evaluate $\int_0^{\pi} \frac{\sin^4 \theta}{a + b\cos\theta} d\theta$ where $a > b > 0$. 7

(b) By method of contour integration, prove that : 7

$$\int_{-\infty}^{\infty} \frac{x^2 - x + 2}{x^4 + 10x^2 + 9} . dx = \frac{5\pi}{12}$$

8. (a) Prove that at each point z of a domain where $f(z)$ is analytic and $f'(z_0) = 0$, z_0 being an interior point, the mapping $w = f(z)$ is conformal. 7

(b) State and prove Hurwitz's Theorem. 7

SECTION – V

9. (a) Define single valued function and multivalued function. $2 \times 7 = 14$

(b) For what value of z, the function w defined by $z = e^{-v}(\cos u + i \sin u)$ ceases to be analytic.

(c) State Cauchy elementary theorem.

(d) Using Cauchy's integral formula, prove that :

$$\int_C \frac{e^z}{z - \pi i} dz = -2\pi i,$$

where C is the circle $|z| = 4$.

(e) Show that e^{-1/z^2} has no singularity.

(f) State Jordan lemma.

(g) Find all points where the mapping $f(z) = \sin z$ is conformal.