Roll No. .....

86091

Master of Science (Mathematics) 1st Sem. Examination – December, 2024

ABSTRACT ALGEBRA

Paper : 24MAT201DS01

Time : Three Hours] *[Maximum Marks : 70*

Before answering the questions, candidates should ensure that they have been supplied the correct and complete question paper. No complaint in this regard, will be entertained after examination.

Note : Attempt ***five*** questions in all, selecting ***one*** question from each Section. Question No. **1** is ***compulsory***. All questions carry equal marks.

1. ***Compulsory*** Question :

(i) Show that a group of order 28 is not simple. 2

(ii) Prove that S_4 is solvable. 2

(iii) How can we find order of a double coset ? 1

(iv) Prove that centre of a nilpotent group G is always non-trivial. 2

(v) Give equivalent conditions for a Noetherian ring. 2

(vi) Describe opposite rings. 1

(vii) State fundamental structure theorem of finitely generated modules over principal ideal domain. 2

(viii) Prove that the kernel of a homomorphism is a submodule. 2

SECTION – I

2. (a) Let G be a finite group of order p^n, where p is a prime. Show that G has subgroups of order 1, p, p^2, ... p^n. 7

(b) Prove that any two Sylow p-subgroups of a finite group G are conjugates in G. 7

3. (a) State and prove Sylow's third Theorem. 7

(b) Let G be a finite group such that, $o(G) = p^n$ where p is a prime. Prove that any subgroup of order p^{n-1} is a normal subgroup of G. 7

SECTION – II

4. (a) Prove that a group G is solvable iff $G^{(n)} = < e >$ for some $n \geq 0$. 7

(b) Let G be a group and H be a normal subgroup of G. Then, if H and G/H both are solvable then prove that G is also a solvable group. 7

5. (a) If H is a central subgroup of G. Also, $H \triangleq G$, both H and G/H are nilpotent subgroups of G. Then, prove that G must also nilpotent. 7

(b) State and prove Schreier's Refinement Theorem. 7

SECTION – III

6. (a) State and prove Schur's lemma. 7

(b) Prove that Q is not a free Z-module. 7

7. (a) Prove that let R be a ring with unity and M is a free R module with basis $(e_1, e_2, \ldots\ldots, e_n)$, then $M \cong R^n$. 7

(b) Prove that if N is a finitely generated free module over a commutative ring R, then all its bases are finite. 7

SECTION – IV

8. (a) Prove that direct sum of Noetherian modules is Noetherian. 7

(b) Let M be a left R-module and N be a submodule of M, then prove that M is Artinian iff N and M/N both are Artinian. 7

9. (a) State and prove Wedderburn Artin theorem. 7

(b) Let R be a left Artinian ring with unity having no nonzero nilpotent ideals, then prove that every nonzero left ideal of R contains nonzero idempotents. 7