24018

B. Tech Common for all branches 2nd Semester

F. Scheme Examination,

May–2015

MATHEMATICS–II

Paper–MATH–102–F

Time allowed : 3 hours] *[Maximum marks : 100*

Note : ***Question No. 1 is compulsory.*** *Attempt total **five** questions with selecting **one** question from each Unit. All questions carry equal marks.*

1. (a) State Stoke's theorem. $2\frac{1}{2}\times 8$

(b) Write inverse L. T. of $\frac{s}{\left(s^2+a^2\right)^2}$

(c) Solve $(x^2 - ay)\,dx - (ax - y^2)\,dy = 0$

(d) Find unit vector normal to the surface $\phi = x^2 yz + xy^3 - 6\,xyz$ at the point $(1, 2, -1)$

(e) Find the Orthogonal trajectories of $y^2 = 4\,ax$.

(f) Find the P. I. of $(D^3 + 1)\,y = \sin(2x + 3)$.

(g) Find the Laplace transform of $\frac{e^{-t}\sin t}{t}$.

(h) Solve $z = px + qy + \sqrt{pq}$

Unit–1

2. (a) Find the directional derivative of $\nabla \cdot (\nabla f)$ at the point (1, –2, 1) in the direction of the normal to the surface $xy^2 z = 3x + z^2$, where $f = 2x^3 y^2 z^4$

(b) (i) Prove that

$$\nabla \times (F \times G) = (\nabla . G) F - (\nabla . F) G + (G . \nabla) F - (F . \nabla) G$$

(ii) If $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$, and

$r \neq 0$ show that

$$\nabla^2 (r^n) = n(n+1) r^{n-2}$$

3. (a) Verify Green's theorem in the plane for $\int_C [2y^2 dx + 3xdy]$,

where C is the boundary of the closed region bounded between $y = x$ and $y = x^2$.

(b) Verify divergence theorem for $\vec{F} = 4x\hat{i} - 2y^2\hat{j} + z^2\hat{k}$ over the region bounded by the cylinder $x^2 + y^2 = 4, z = 0, z = 3$.

Unit–2

4. (a) Solve the following differential equation :

$$(xy^2 + 2x^2y^3) dx - (x^3y^2 - x^2y) dy = 0$$

(b) When a resistance of R ohms is connected in series with an inductance of L henries with an emf of E volts, the current i amperes at time t is given by $L\frac{di}{dt}+Ri=E$.

If $E=10\sin t$ volts and $i=0$ when $t=0$, find i as a function of t.

5. (a) Solve :

$$(3x+2)^2\frac{d^2y}{dx^2}+3(3x+2)\frac{dy}{dx}-36y=3x^2+4x+1.$$

(b) An electric circuit consists of an inductance of 0.1 Henry, a resistance of $20\,\Omega$ and a condenser of capacitance 25 micro farads. Find the charge q and the current i at any given time t, given that at $t=0,\ q=0.05c,\ i=\frac{dq}{dt}=0$, when $t=0$.

Unit–3

6. (a) Find the Laplace transform of

(i) sinh at cos at

(ii) Evaluate $\int_0^\infty t\sin 3t\,dt$.

(b) Apply convolution theorem to evaluate

$$L^{-1}\left\{\frac{1}{(s^2+1)(s^2+9)}\right\}$$

7. (a) (i) Find Laplace transform of f (t), where

$f(t) = |t-1| + |t+1|,\ t > 0.$

(ii) Find Laplace Inverse of $\log\left(\frac{s+a}{s+b}\right)$

(b) Solve $\frac{d^2x}{dt^2} + 2\frac{dx}{dt} + 5x = e^{-t}\sin t$

given that $x(0) = 0$ and $x'(0) = 1$ by using Laplace transform.

Unit–4

8. (a) Solve the following differential equation

$x^2(y-z)p + y^2(z-x)q = z^2(x-y)$

(b) Solve the equation by Charpit's method

$2z + p^2 + qy + 2y^2 = 0.$

9. (a) Form partial differential equation from the relation

$z = y^2 + 2f\left(\frac{1}{x} + \log y\right).$

(b) Using method of separation of variables,

$4(\partial u/\partial x) + (\partial u/\partial y) = 3u$, given

$u = 3e^{-y} - e^{-5y}$ when $x = 0$