7. (a) Find the maximum and minimum distances of the point (3,4,12) from the sphere $x^2+y^2+z^2=1$. 7.5

(b) Find the directional derivative of the $f=xy^2+yz^2$ at the point $(2,-1,1)$ in direction of the vector $\hat{i}+2\hat{j}+2\hat{k}$. 7.5

UNIT – IV

8. (a) Find the Inverse of Matrix : 7.5

$$A=\begin{bmatrix}2 & 1 & -1\\ 0 & 2 & 1\\ 5 & 2 & -3\end{bmatrix}$$

using Elementary transformations.

(b) Find the values of a and b for which the equations $x+ay+z=3, x+2y+2z=b$ $x+5y+3z=9$ are consistent. When will these equations have unique solution ? 7.5

9. (a) Find the Eigen values and Eigen vectors of Matrix : 7.5

$$A=\begin{bmatrix}-2 & 2 & -3\\ 2 & 1 & -6\\ -1 & -2 & 0\end{bmatrix}$$

(b) Verify Cayley-Hamilton theorem and find the Inverse of matrix : 7.5

$$A=\begin{bmatrix}1 & 1 & 3\\ 1 & 3 & -3\\ -2 & -4 & -4\end{bmatrix}$$

Roll No. ....

3007

B. Tech. 1st Semester (ME)

Examination – December, 2018

MATHEMATICS - I (CALCULUS & MATRICES)

Paper : BSC-Math-101-G

Time : Three Hours] *[Maximum Marks : 75*

Before answering the questions, candidates should ensure that they have been supplied the correct and complete question paper. No complaint in this regard, will be entertained after examination.

Note : Attempt *five* questions in all, selecting at least *one* question from each Unit. Question No. 1 is *compulsory*. All questions carry equal marks.

1. (a) Determine Rank of Matrix. : $6\times 2.5=15$

$$A=\begin{bmatrix}2 & 5 & 6\\ 1 & 2 & 3\\ 3 & 7 & 9\end{bmatrix}$$

(b) Evaluate the following limit

$$f(x,y)=\underset{x\to 1}{Lt}\frac{2x^2y}{x^2+y^2+1}$$

(c) If $z=\sin^{-1}\left[\frac{x^2+y^2}{x-y}\right]^{y\to 2}$ Determine $\left(\frac{\partial z}{\partial y}\right)$.

(d) State the Result of Lagranges's mean value theorem.

(e) Compute $B\left(\frac{5}{2},\frac{3}{2}\right)$.

(f) Evaluate Curl $\left[e^{xyz}(i+j+k)\right]$.

UNIT – I

2. (a) Evaluate : 8

(i) $\underset{x\to 1}{Limit}\frac{x^x-x}{x-1-\log x}$

(ii) $\underset{x\to\frac{\pi}{2}}{Lt}(\tan x)^{\cos x}$

(b) Show that the evolute of the cycloid $x=a(\theta-\sin\theta)$, $y=a(1-\cos\theta)$ is another equal cycloid. 7

3. (a) Find the volume of the solid generated by revolving the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ about y-axis. 7

(b) Prove that :

(i) $\int_0^1\frac{x\,dx}{\sqrt{1-x^5}}=\frac{1}{5}B\left(\frac{2}{5},\frac{1}{2}\right)$ 8

(ii) $\int_0^1\frac{dx}{\sqrt{1+x^4}}=\frac{1}{4\sqrt{2}}B\left(\frac{1}{4},\frac{1}{2}\right)$

UNIT – II

4. (a) Test the convergence of following series : 7.5

$$1+\frac{x}{2}+\frac{x^2}{5}+\frac{x^3}{10}+......+\frac{x^n}{n^2+1}+......$$

(b) Test for convergence the series : 7.5

$$\sum\frac{4.7.........(3n+1)x^n}{1.2.3.4.\ ....n}$$

5. (a) Test the convergence of an Alternating series : 7.5

$$\frac{1}{1.2}-\frac{1}{3.4}+\frac{1}{5.6}-\frac{1}{7.8}+.......$$

(b) Find a Fourier series of $f(x)=x^2$ over the interval. $[-\pi,\ \pi]$. 7.5

UNIT – III

6. (a) If $v=\left(x^2+y^2+z^2\right)^{-1/2}$ then prove that $\frac{\partial^2 v}{\partial x^2}+\frac{\partial^2 v}{\partial y^2}+\frac{\partial^2 v}{\partial z^2}=0$. 7.5

(b) If $z=f(x,y)$ and $x=e^u+e^{-v}, y=e^{-u}-e^v$ prove that $\frac{\partial z}{\partial u}-\frac{\partial z}{\partial v}=x\frac{\partial z}{\partial x}-y\frac{\partial z}{\partial y}$. 7.5