Roll No. .....

24002

B. Tech 1st Semester (Common for All Branches) Examination – December, 2017

MATHEMATICS-I

Paper : Math-101-F

Time : Three Hours] *[Maximum Marks : 100*

Before answering the questions, candidates should ensure that they have been supplied the correct and complete question paper. No complaint in this regard, will be entertained after examination.

Note : Attempt total ***five*** questions, selecting ***one*** question from each Sections. Question No. **1** is ***compulsory***. All questions carry equal marks.

1. (a) Test the convergence of :

$$\sum_{n=1}^{\infty}\left(\frac{1}{\sqrt{n}+\sqrt{n+1}}\right).$$

(b) For what values of λ and μ do the equations $x + y + z = 6$, $x + 2y + 3z = 10$, $x + 2y + \lambda z = \mu$ have unique solution.

SECTION – D

8. (a) Find the volume of a spherical cap of height h cut off from a sphere of radius a.

(b) Find, by double integration, the volume generated by revolving the cardioid $r = a(1 + \cos\theta)$ about the initial line.

9. (a) Evaluate $\iint r^3 dr d\theta$, over the area bounded between the circles $r = 2\cos\theta$ and $r = 4\cos\theta$.

(b) By changing the order of integration, evaluate

$$\int_0^{\infty}\int_0^{x} xe^{-\frac{x^n}{y}}\,dydx\,.$$

(c) If $x = r\cos\theta, y = r\sin\theta$ find $\frac{\partial^2 r}{\partial x^2}$ and $\frac{\partial^2 \theta}{\partial y^2}$.

(d) Define Beta and Gamma functions. Also give relationship between them.

SECTION – A

2. (a) Discuss the convergence of the series :

$$\frac{x}{2\sqrt{3}} + \frac{x^2}{3\sqrt{4}} + \frac{x^3}{4\sqrt{5}} + \frac{x^4}{5\sqrt{6}} + \ldots\ldots\infty.$$

(b) Test the convergence of the series :

$$\sum_{n=1}^{\infty} \frac{n!}{(n+1)^n} x^n$$

3. Test $\sum_{n=2}^{\infty} \frac{(-1)^n}{n(\log n)^2}$ for convergence and absolute convergence.

SECTION – B

4. (a) Find the rank of the matrix :

$$\begin{bmatrix} 1 & 2 & 0 & -1 \\ 3 & 4 & 1 & 2 \\ -2 & 3 & 2 & 5 \end{bmatrix}$$

by reducing it in its normal form.

(b) Are the following vectors linearly dependent ? If so, find the relation between them :

$x_1 = (1,2,1), x_2 = (2,1,4), x_3 = (4,5,6), x_4 = (1,8,-3)$.

5. Diagonalise the matrix :

$$A = \begin{bmatrix} 3 & -1 & 1 \\ -1 & 5 & -1 \\ 1 & -1 & 3 \end{bmatrix}$$ and hence find A^4.

SECTION – C

6. (a) Using Taylor's series, expand $\sin x$ in powers of $\left(x - \frac{\pi}{2}\right)$. Hence find the value of sin 91° correct to four decimal places.

(b) Find the radius of curvature for the curve $r = a(1 + \cos\theta)$.

7. (a) Find the maximum and minimum distances of the point (3, 4, 12) from the sphere $x^2 + y^2 + z^2 = 1$.

(b) Evaluate the integral :

$$\int_0^{\pi/2} \frac{\log(1 + a\sin^2 x)}{\sin^2 x} dx.$$