7. (a) If $u = \tan^{-1}\left(\frac{x}{-}\right.$ $\left.-\right)$ prove that

$$x\frac{\partial u}{\partial x} + y = \sin 2u$$

and

$$x^2\frac{\partial^2 u}{\partial x^2} + y \quad + 2xy\frac{\partial^2 u}{\partial x \partial y} = 2\cos 3u \sin u.$$

(b) A rectangul ox open at the top is to have a volum 32 c.c. Find the dimensions of the box r ing least material for its construction.

IV

8. (a) Find the are ng between the parabola $y = 4x - x^2$ an ine $y = x$.

(b) Find, by doubl gration, the volume of the solid generated by ving the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ about the y-ax

9. (a) Find the volu he ellipsoid

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

(b) Prove that

$$\beta(m, n) = \frac{\Gamma n}{\Gamma m}$$ hence evaluate $\Gamma 1/2$.

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B.Tech. 1st Semester F-Scheme Examination,

December–2016

MATHEMATICS–I

Paper–MATH–101–F

Time allowed : 3 hours] *[Maximum marks : 100*

Note : ***Question No. 1 is compulsory.*** *Attempt total* ***five*** *questions with selecting* ***one*** *question from each unit. All questions carry equal marks.*

1. (a) Discuss the behavior of series

$$\frac{\sqrt{2}-1}{3^3-1} + \frac{\sqrt{3}-1}{4^3-1} + \frac{\sqrt{4}-1}{5^3-1} + \frac{\sqrt{5}-1}{6^3-1} + \text{-------}$$

(b) Prove that

$$\log(1+x) = x - \frac{x^2}{2} - \frac{x^3}{3} + \frac{x^4}{4} \text{ ------}$$

(c) Find asymptotes parallel to co-ordinate axis of the curve

$$x^2y^2 - x^2y - xy^2 + x + y + 1 = 0$$

(d) Define rank of matrix. Find rank of matrix

$$\begin{bmatrix} 1 & 2 & 3 & 4 \\ -2 & 0 & 5 & 7 \end{bmatrix}$$

(e) Using Cayley-Hamilton theorem, find A^6 if

$$\begin{bmatrix} 2 & 1 \\ 5 & -2 \end{bmatrix}$$

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(f) Write the relatio… between Cartesian coords and cylindrica… ar coords. Also write relationship be… Cartesian coords and Spherical polar c…

(g) Define Beta and … a function.

(h) State Leibnitz th… for the nth derivative of the product of tw… tions.

U…

2. (a) Examine the co… nce of the series whose nth term is $\sqrt{n^4}$… $\sqrt{n^4-1}$.

(b) Discuss the conv… ce of the series

$$\frac{2}{1}x+\frac{9}{8}x^2+\frac{64}{81}x^3+\ldots\ldots\text{ to }\infty$$

3. (a) Test for absolute/… ionally convergence of the series $2-\frac{4}{2!}+\frac{8}{3}\ldots+\frac{32}{5!}\ldots\ldots$

(b) For what values … are the following series convergent : $\frac{1}{1-x}+\frac{1}{(1-x)^2}+\frac{1}{3(1-x)^3}+\ldots.\infty$

U…

4. (a) Find non-singu… matrices P and Q such that PAQ is in… normal form for the matrix

$$A=\begin{bmatrix}0 & 2 & 4\\ 2 & 3 & 4\\ 4 & 8 & 12\end{bmatrix}$$

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(b) Find the value of λ such that the foll… bwing equations have unique solution :
$\lambda x+2y-2z-1=0,\quad 4x+2\lambda y-z-2=0,$
$6x+6y+\lambda z-3=0$
and use matrix method to solve these equ… ations when $\lambda=2$

5. (a) If λ is an eigen value of a non-singular ma… rix A, show that

(i) $\lambda-1$ is an eigen value of $A-1$

(ii) $\frac{|A|}{\lambda}$ is an eigen value of adj. A

(b) Show that the matrix

$$A=\begin{bmatrix}1 & 2 & 0\\ 2 & -1 & 0\\ 0 & 0 & -1\end{bmatrix}$$

satisfies its own characteristic equation and … hence or otherwise find A^{-1}.

Unit–III

6. (a) If $\log y=\tan^{-1}x$, show that
$(1+x^2)y_{n+2}+\{2(n+1)x-1\}y_{n+1}+n(n+1)y_n=0$
and hence find y_3, y_4 and y_5 at $x=0$.

(b) Show that the radius of curvature ρ at P on an ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ is given by $\rho=\frac{CD^3}{ab}$ where CD is the semi-diameter conju… gate to CP.

(c) Expand $e^x\log(1+y)$ in powers of x and y upto terms of third degree.

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