Roll No. ....

24002

B.Tech. 1st Semester

Examination – December, 2013

MATHEMATICS-I

('F' Scheme)

Paper : Math-101-F

Time : Three hours] *[Maximum Marks : 100*

Before answering the questions, candidates should ensure that they have been supplied the correct and complete question paper. No complaint in this regard, will be entertained after examination.

Note : Question 1st is ***compulsory***. Attempt total *five* questions with selecting *one* question from each Unit. All questions carry equal marks.

1. (a) What will be the product of Eigen values of singular matrix ?

   (b) Using cayley-Hamilton theorem, find A^4 if

$$\begin{bmatrix} 2 & 1 \\ 5 & -2 \end{bmatrix}$$

(c) Discuss the behavior of series

$$\frac{\sqrt{2}-1}{3^3-1}+\frac{\sqrt{3}-1}{4^3-1}+\frac{\sqrt{4}-1}{5^3-1}+\frac{\sqrt{5}-1}{6^3-1}+------$$

(d) Find asymptotes parallel to co-ordinate axis of the curve

$$x^2y^2-x^2y-xy^2+x+y+1=0$$

(e) Expand α^x by using maclaurin's series

(f) If $u=\tan^{-1}\left(\frac{x^3+y^3}{x-y}\right)$ then prove

$$x\frac{\partial u}{\partial x}+y\frac{\partial u}{\partial y}=\sin 2u.$$

(g) Write the formula of finding the volume of the solid generated by revolving about both the co-ordinate axis.

(h) State p-Test for convergence of series.

UNIT – I

2. (a) Discuss the convergence of the series :

$$1+\frac{\alpha+1}{\beta+1}+\frac{(\alpha+1)(2\alpha+1)}{(\beta+1)(2\beta+1)}+\frac{(\alpha+1)(2\alpha+1)(3\alpha+1)}{(\beta+1)(2\beta+1)(3\beta+1)}+.....\infty$$

(b) $x+\frac{(2x)^2}{2!}+\frac{(3x)^3}{3!}+\frac{(4x)^4}{4!}+$..............................

3. (a) Test for absolute/conditionally convergence of the series.

$$\frac{1}{6}x^2-\frac{2}{11}x^3+\frac{3}{16}x^4-\frac{4}{21}x^5+\frac{5}{26}x^6..............$$

(b) Discuss the convergence of the series

$$\frac{1}{2}+\frac{2}{3}x+\left(\frac{3}{4}\right)^2x^2+\left(\frac{4}{5}\right)^3x^3+...\infty,$$

UNIT – II

4. (a) Find the characteristic roots and characteristic vectors of the matrix.

$$A = \begin{bmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & 3 \end{bmatrix}$$

(b) Find non-singular matrices P and Q such that PAQ is in the normal form for the matrices

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 3 & 2 & 1 \\ 1 & 3 & 2 \\ 2 & 1 & 3 \end{bmatrix}$$ Also find the rank of matrix A

5. (a) Find the characteristic equation of the matrix

$$A = \begin{pmatrix} 1 & 3 & 7 \\ 4 & 2 & 3 \\ 1 & 2 & 1 \end{pmatrix}$$ Hence obtain the inverse of the given matrix.

(b) Are the following vectors are linearly dependent ?

If so, find the relation between them.

$$x_1 = (1,2,1), x_2 = (2,1,4), x_3 = (4,5,6), x_4 = (1,8,-3)$$

UNIT – III

6. (a) If $u = \sin^{-1}\left(\frac{x+y}{\sqrt{x}-\sqrt{y}}\right)$

then prove that

$$x^2 \frac{\partial^2 u}{\partial x^2} + y^2 \frac{\partial^2 u}{\partial y^2} + 2xy \frac{\partial^2 u}{\partial x \partial y} = \frac{-\cos 2u \sin u}{4 \cos^3 u}$$

(b) Discuss the maxima and minima of

$\sin x + \sin y + \sin (x + y)$

(c) By using Taylor's series prove that

$\sin (x + y) = \sin x \cos y + \cos x \sin y$.

Hence find the value of $\sin 46^0$ correct to four places of decimals. 8,6,6

7. (a) If ρ_1 & ρ_2 be the radii of curvature at the extremities of a focal chord of a parabola whose latus rectum is '4a' then prove that

$$\rho_1^{-2/3} + \rho_2^{-2/3} = 2a^{-2/3}$$

(b) Prove that $\int_0^\infty \frac{\tan^{-1} ax}{(x)(1+x^2)} dx = \frac{\pi}{2} \log(1+a)$, where $a > 0$.

UNIT – IV

8. (a) Evaluate by changing the order of integration

$$\int_0^1 \int_x^{\sqrt{2-x^2}} \frac{x\,dy\,dx}{\sqrt{x^2+y2}}$$

(b) Find the double integration, the area lying inside the circle $r = a \sin \theta$ and outside the cardiod $r = a(1 - \cos \theta)$.

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9. (a) Find the volume of the ellipsoid

$$\frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2}=1$$

(b) Prove that :

$$\beta(m,n)=\frac{\Gamma m\,\Gamma n}{\Gamma m+n}, \text{ hence evaluate } \Gamma 1/2.$$

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