

Time allowed : 3 hours] [Maximum marks : 75

Note : Attempt five questions in total, first being compulsory and selecting one from each unit.

1. (a) Write a short note on
- Probability spaces
 - Independent random variables
- (b) A Community consists of 50 percent Hindus, 30 percent Muslims and 20 percent Sikhs. If a sample of six individuals is selected at random, what is the probability that two are Hindus, three are Muslims and one is a Sikh?
- (c) The response time of a certain computer system in seconds has an exponential distribution with a mean of 3 seconds what is the probability that response time exceeds 5 seconds?
- (d) Determine the mean of a Binomial distribution
- (e) The two regression equations of the variables x and y are $x = 19.13 - 0.87y$ and $y = 11.64 - 0.5x$. Find \bar{x} and \bar{y} .
- (f) A random sample of 15 paired observations from a bivariate normal population gives a correlation coefficient of 0.5. Does this signify the existence of correlation in the sampled population? 15

Unit-I

2. (a) A purse contains 2 silver and 4 copper coins and a second purse contains 4 silver and 4 copper coins. If a coin is selected at random from one of the two purses, what is the probability that it is a silver coin?
- (b) A variate X has the probability distribution
- | | | | |
|----------------|-----|-----|-----|
| $x :$ | -3 | 6 | 9 |
| $P\{x = x\} :$ | 1/6 | 1/2 | 1/3 |
- Evaluate (i) $E(2x + 1)^2$ (ii) $\text{Var}(4x + 5)$
3. (a) State and prove Chebyshev's inequality 15
- (b) A manufacturer knows that the condensers he makes contains on the average 1% defective. He packs them in the boxes of 100. What is the probability that a box picked at random will contain 3 or more faulty condensers? 15

Unit-II

4. (a) If X is a continuous random variable with p.d.f. given by

$$f(x) = \begin{cases} kx, & 0 \leq x < 1 \\ k, & 1 \leq x < 2 \\ -kx + 3k, & 2 \leq x < 3 \\ 0 & \text{otherwise} \end{cases}$$

- Determine (i) The constant k
(ii) c.d.f. $f(x)$

- (b) The mean and standard deviation of the marks obtained by 1000 students in an examination are respectively 34.4 and 16.5. Assuming the normality of the distribution, find the approximate number of students expected to obtain marks between 30 and 60. 15

5. (a) If X and Y are two independent continuous random variables, then determine the p.d.f. of the random variable $U = X + Y$.
- (b) The joint p.d.f. of a two-dimensional random variable (x, y) is given by

$$f(x, y) = \begin{cases} 2; & 0 < x < 1, \quad 0 < y < x \\ 0 & \text{elsewhere} \end{cases} \quad 15$$

Find the conditional density function of Y given $X = x$ and conditional density function of X given $Y = y$.

Unit-III

6. (a) Calculate mean of the following data

Class interval	0-10	10-20	20-30	30-40	40-50	50-60
Frequency (f)	14	17	22	26	23	18

- (b) The first four moments about the working mean 28.5 of a distribution are 0.294, 7.144, 42.409 and 454.98. Calculate the moments about the mean. Also comment upon the skewness and Kurtosis of the distribution. 15
7. (a) Find correlation coefficient between X and Y from the given data

X	55	56	58	59	60	60	62
Y	35	38	38	39	44	43	45

- (b) Fit a second degree parabola to the following data. 15

X	0	1	2	3	4
Y	1	1.8	1.3	2.5	6.3

[P. T. O.]

Unit-IV

8. (a) A coin was tossed 400 times and head turned up 216 times. Test the hypothesis that the coin is unbiased at 5% level of significance.
- (b) A simple sample of heights of 6400 Englishmen has a mean of 170cm and a s.d. of 6.4cm. While a simple sample of heights of 1600 Americans has a mean of 172cm and a s.d. of 6.3cm. Does the data indicate that Americans are, on the average, taller than Englishmen? 15
9. (a) Two random samples gave the following results

Sample	Size	Sample Mean	Sum of squares of deviations from mean
1	10	15	90
2	12	14	108

Test whether the sample came from the sample normal population.

- (b) 200 digits are chosen at random from a set of tables. The frequencies of the digits are as follows.

Digit	0	1	2	3	4	5	6	7	8	9
Frequency	18	19	23	21	16	25	22	20	21	15

Using X^2 - test to assess the correctness of hypothesis that digits were distributed in equal number in tables from where they are chosen. 15