

24018

B.Tech. 2nd Semester F-Scheme Examination,
May-2018

MATHEMATICS-II

Paper-MATH-102-F

(Common for All Branches)

Time allowed : 3 hours] [Maximum marks : 100

Note : Question No. 1 is compulsory. Attempt total five questions with selecting one question from each unit. All questions carry equal marks.

1. (a) If $\vec{F} = \text{grad}(x^3 + y^3 + z^3 - 3xyz)$. Find Div $\vec{F}$ and curl $\vec{F}$
- (b) Show that, if $R = \sin \omega t + B \cos \omega t$, where A, B, ω are constants, then

$$\frac{d^2 R}{dt^2} = -\omega^2 R \text{ and } R \times \frac{dR}{dt} = -\omega \times R$$

- (c) Solve $pq = p + q$.

- (d) Find Laplace transformation of $\frac{e^{-t} \sin t}{1}$

- (e) Solve $\frac{\partial^2 z}{\partial y \partial x} = xy$

- (f) Solve $\frac{d^2 y}{dx^2} + \frac{dy}{dx} = e^{-x}$

(2)

24018

- (g) Find the inverse Laplace transformation of $\frac{s+3}{s^2+6s+13}$
- (h) Solve $(2x^3 - xy^2 - 2y + 3) dx - (x^2y + 2x) dy = 0$.

Unit-I

2. (a) Give the geometrical interpretation of gradient, also prove that $\text{curl curl } \vec{F} = \text{grad}(\text{div } \vec{F}) - \nabla^2 \vec{F}$.
- (b) Prove that
- (i) $\nabla \times (\vec{F} \times \vec{G}) = (\nabla \cdot \vec{G}) \vec{F} - (\nabla \cdot \vec{F}) \vec{G} + (\vec{G} \cdot \nabla) \vec{F} - (\vec{F} \cdot \nabla) \vec{G}$
- (ii) $\nabla^2 f(r) = f''(r) + \frac{2}{r} f'(r)$.
3. (a) Verify Green's theorem in the plane for $\oint_C [(x^2 + xy) dx + (x^2 + y^2) dy]$, where C is the square formed by the lines $y = \pm 1, x = \pm 1$.
- (b) Verify Stoke's theorem for the vector field $\vec{F} = (2x - y) \mathbf{i} - yz^2 \mathbf{j} - y^2 z \mathbf{k}$ over the upper half surface of $x^2 + y^2 + z^2 = 1$ bounded by its projection on the xy -plane.

Unit-II

4. (a) Solve the following differential equation :

$$(2x^2y - 3y^4) dx + (3x^3 + 2xy^3) dy = 0$$

- (b) A constant e.m.f. E volts is applied to a circuit containing a constant resistance R ohms in series and a constant inductance L henries. If the initial current is zero, show that the current builds up to half its theoretical maximum is $(L \log 2)/R$ seconds.

5. (a) By the method of variation of parameters :

$$\text{solve } \frac{d^2y}{dx^2} + 4y = \tan 2x.$$

- (b) Solve the equation

$$(1+x)^2 \frac{d^2y}{dx^2} + (1+x) \frac{dy}{dx} + y = 4 \cos \log(1+x).$$

Unit-III

6. (a) Find the Laplace transform of

(i) $\cosh at, \cos at$

(ii) $\int_0^1 t \sin 3t dt$

- (b) Find inverse Laplace transformation of $(se^{-t/2} + \pi e^{-t}) / (s^2 + \pi^2)$.

7. (a) Using Convolution theorem, find Inverse Laplace transformation of $\frac{s}{(s^2 + a^2)^2}$

- (b) Solve

$$\frac{d^2x}{dt^2} + \frac{dx}{dt} + 5x = e^{-t} \sin t, x(0) = 1, x'(0) = -1,$$

using Laplace transform.

Unit-IV

8. (a) Solve the following differential equation

$$(y+z)p - (z+x)q = (x-y)$$

- (b) Solve the equation by Charpit's method

$$(p^2 + q^2)y = qz$$

9. (a) Find the differential equation of all planes which are at a constant distance 'a' from the origin.

- (b) Using method of separation of variables,

$$\text{solve } \frac{\partial u}{\partial x} = 2 \frac{\partial u}{\partial t} + u, \text{ where } u(x, 0) = 6e^{-3x}.$$