

24018

B.Tech. 2nd Semester F Scheme

Examination, May-2014

MATHEMATICS-II

Paper-Math-102-F

Common for all branches

Time allowed : 3 hours] [Maximum marks : 100

Note : Question No. 1 is compulsory. Attempt five questions in total.

1. (a) Define constant vector.
- (b) Find value of curl (grad ϕ).
- (c) Solve $(3x^2 + 6xy^2) dx + (6x^2y + 4y^3) dy = 0$.
- (d) Solve $\frac{d^2x}{dt^2} + 6 \frac{dx}{dt} + 9x = 0$.
- (e) Find the Laplace transform of $e^{-3t} \cos^2 t$.
- (f) Define Laplace transform of periodic function.
- (g) Solve $\frac{\partial^2 u}{\partial x \partial t} = e^{-t} \cos x$.
- (h) Solve $z = px + qy - 2 \sqrt{pq}$.

Section-A

2. (a) Calculate the angle between the normals to the surface $xy = z^2$ at the points (4, 1, 2) and (3, 3, -3).

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[P.T.O.]

- (b) A vector field is given by

$\vec{A} = (x^2 + xy^2) \hat{i} + (y^2 + x^2y) \hat{j}$. Show that the field is irrotational and find the scalar potential.

3. (a) Find the circulation of $\vec{F}$ round the curve c , where $\vec{F} = e^x \sin y \hat{i} + e^x \cos y \hat{j}$ and c is the rectangle whose vertices are

$$(0, 0), (1, 0), \left(1, \frac{\pi}{2}\right) \text{ and } \left(0, \frac{\pi}{2}\right).$$

- (b) Verify Stoke's theorem for the vector field $\vec{F} = (2x - y) \hat{i} - yz^2 \hat{j} - y^2z \hat{k}$ over the upper half surface of $x^2 + y^2 + z^2 = 1$, bounded by its projection on the xy -plane.

Section-B

4. (a) Solve $y(xy + 2x^2y^2) dx + x(xy - x^2y^2) dy = 0$.
- (b) When a switch is closed in a circuit containing a battery E , a resistance R and an inductance L , the current i builds up at a rate given by $L \frac{di}{dt} + Ri = E$. Find i as a function of t . How long will it be, before the current has reached one half its maximum value if $E = 6$ volts, $R = 100$ ohms and $L = 0.1$ henry ?

5. (a) Apply the method of variation of parameters to

solve $\frac{d^2y}{dx^2} - y = \frac{2}{1+e^x}$.

(b) Solve $x^2 \frac{d^2y}{dx^2} + 3x \frac{dy}{dx} + y = \frac{1}{(1-x)^2}$.

Section-C

6. (a) Find the inverse Laplace transform of

$$\frac{1}{s^3(s^2 + a^2)}.$$

- (b) Find the inverse Laplace transform of

$$\cot^{-1}\left(\frac{s+a}{b}\right).$$

- (c) Apply convolution theorem to evaluate

$$L^{-1}\left[\frac{s^2}{(s^2+4)^2}\right].$$

7. (a) Solve $\frac{d^2y}{dt^2} + \frac{dy}{dt} - 2y = 3 \cos 3t - 11 \sin 3t$ given

that $y(0) = 0$ and $y'(0) = 6$ by Laplace transform.

- (b) Solve $\int_0^t \frac{y(u)}{\sqrt{t-u}} du = \sqrt{t}$ by Laplace transform.

Section-D

8. (a) Form the partial differential equation by eliminating the arbitrary functions from $f(x^2 + y^2 + z^2, z^2 - 2xy) = 0$.
- (b) Solve $x^2(y - z)p + y^2(z - x)q = z^2(x - y)$.
- (c) Solve $2zx - px^2 - 2qxy + pq = 0$.
9. (a) Solve the equation $\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2}$ with boundary conditions
 $u(x, 0) = 3 \sin n\pi x, u(0, t) = 0, u(1, t) = 0$,
 where $0 < x < 1, t > 0$.
- (b) Solve $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$ within the rectangle
 $0 \leq x \leq a, 0 \leq y \leq b$ given that
 $u(0, y) = u(a, y) = u(x, b) = 0$ and
 $u(x, 0) = x(a - x)$.