24018

B.Tech. 2nd Semester F-Scheme

(Common for All Branches) Examination,

May-2019

MATHEMATICS-II

Paper-Math-102-F

Time allowed : 3 hours] *[Maximum marks : 100*

Note : *Attempt **five** questions in total selecting **one** question from each section. **Question No. 1 is compulsory.***

1. (a) Give physical interpretation of gradient and divergence.

   (b) Solve $(D^2 + 4)\,y = \cos 2x$.

   (c) find the Laplace transform of $e^{-3t}\cos^2 t$.

   (d) Form the partial differential equation by eliminating the function f from the relation

$$z = y^2 + 2\,f\left(\frac{1}{x} + \log y\right)$$

Section-A

2. (a) Find the values of constants a, b and c so that the maximum value of the directional derivative of :

$$\phi = axy^2 + 6yz + cz^2x^3 \text{ at } (1, 2, -1)$$

has a magnitude 64 in the direction parallel to z–axis.

   (b) prove that $(y^2 - z^2 + 3yz - 2x)\hat{i} + (3xz + 2xy)\hat{j} + (3xy - 2xz + 2z)\hat{k}$ is both solenoidal and irrotational.

3. (a) Verify divergence theorem for $\vec{F} = xz\hat{i} + y^2\hat{j} + yz\hat{k}$ taken over the cube bounded by $x = 0, x = 1 ; y = 0, y = 1; z = 0, z = 1$.

Section-B

4. (a) Solve $(xy^3 + y)\,dx + 2(x^2y^2 + x + y^4) = 0$.

   (b) If the air is maintained at 30°C and the temperature of the body cools from 80°C to 60°C in 12 minutes, find the temperature of the body after 24 minutes.

5. (a) solve $\frac{d^2y}{dx^2} + 4y = \tan 2x$ by method of variation of parameters.

   (b) Solve $x^2\frac{d^2y}{dx^2} + x\frac{dy}{dx} + y = \log x \sin(\log x)$

Section-C

6. (a) Find the inverse transform of $\log\left(\frac{s(s+1)}{s^2+4}\right)$.

(b) Find the inverse transform of $\frac{1}{(s^2+a^2)^2}$ by applying convolution theorem.

7. (a) Solve the equation :

$y'' - 3y' + 2y = 4t + e^{3t}$, when $y(0) = 1$, $y'(0) = -1$

by using Laplace transform.

(b) Solve the integral equation by Laplace transform:

$$\int_0^t \frac{y(u)}{t-u}\, du = 1 + t + t^2$$

Section-D

8. (a) Solve the Lagrange linear equation :

$x(y-z)\, p + y\, (z-x)\, q = z(x-y)$.

(b) Solve $xp + 3yq = 2\,(z - x^2q^2)$ by Charpit's method.

9. Solve the equation $\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2}$ with boundary conditions :

$u(x, 0) = 3 \sin n\pi x$, $u(0, t) = 0$

$u(1,t) = 0$, where $0 < x < 1$, $t > 0$.