

7. (a) If  $u = \tan^{-1} \left( \frac{y}{x} \right)$  prove that

$$x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y} = \sin 2u$$

and

$$x^2 \frac{\partial^2 u}{\partial x^2} + y^2 \frac{\partial^2 u}{\partial y^2} + 2xy \frac{\partial^2 u}{\partial x \partial y} = 2 \cos 3u \sin u.$$

- (b) A rectangular box open at the top is to have a volume of 32 c.c. Find the dimensions of the box requiring least material for its construction.

IV

8. (a) Find the area between the parabola  $y = 4x - x^2$  and the line  $y = x$ .  
(b) Find, by double integration, the volume of the solid

generated by revolving the ellipse  $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$  about the y-axis.

9. (a) Find the volume of the ellipsoid

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$$

- (b) Prove that

$$\beta(m, n) = \frac{\Gamma(n)}{\Gamma(m)} \quad \text{hence evaluate } \Gamma(1/2).$$

B.Tech. 1st Semester F-Scheme Examination,

December-2016

MATHEMATICS-I

Paper-MATH-101-F

Time allowed : 3 hours ]

[ Maximum marks : 100

**Note :** Question No. 1 is compulsory. Attempt total five questions with selecting one question from each unit. All questions carry equal marks.

1. (a) Discuss the behavior of series

$$\frac{\sqrt{2}-1}{3^3-1} + \frac{\sqrt{3}-1}{4^3-1} + \frac{\sqrt{4}-1}{5^3-1} + \frac{\sqrt{5}-1}{6^3-1} + \dots$$

- (b) Prove that

$$\log(1+x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \dots$$

- (c) Find asymptotes parallel to co-ordinate axis of the curve

$$x^2 y^2 - x^2 y - xy^2 + x + y + 1 = 0$$

- (d) Define rank of matrix. Find rank of matrix

$$\begin{bmatrix} 1 & 2 & 3 & 4 \\ -2 & 0 & 5 & 7 \end{bmatrix}$$

- (e) Using Cayley-Hamilton theorem, find  $A^6$  if

$$\begin{bmatrix} 2 & 1 \\ 5 & -2 \end{bmatrix}$$

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- (f) Write the relation between Cartesian coordinates and cylindrical coordinates. Also write the relation between Cartesian coordinates and spherical coordinates.
- (g) Define Beta and Gamma functions.
- (h) State Leibnitz theorem for the nth derivative of the product of two functions.

U

2. (a) Examine the convergence of the series whose nth term is  $\sqrt{n^4 - 1}$ .
- (b) Discuss the convergence of the series  $\frac{2}{1}x + \frac{9}{8}x^2 + \frac{64}{81}x^3 + \dots$  to  $\infty$ .
3. (a) Test for absolute/conditional convergence of the series  $2 - \frac{4}{2!} + \frac{8}{3!} - \frac{32}{5!} + \dots$ .
- (b) For what values of x are the following series convergent:  $\frac{1}{1-x} + \frac{1}{3(1-x)^3} + \dots$ .
4. (a) Find non-singular matrices P and Q such that PAQ is in normal form for the matrix

$$A = \begin{bmatrix} 0 & 2 & 4 \\ 2 & 3 & 4 \\ 4 & 8 & 12 \end{bmatrix}$$

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(3)

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- (b) Find the value of  $\lambda$  such that the following equations have unique solution:
- $$\lambda x + 2y - 2z - 1 = 0, \quad 4x + 2\lambda y - z - 2 = 0, \quad 6x + 6y + \lambda z - 3 = 0$$
- and use matrix method to solve these equations when  $\lambda = 2$ .

5. (a) If  $\lambda$  is an eigen value of a non-singular matrix A, show that
- (i)  $\lambda - 1$  is an eigen value of  $A - I$
- (ii)  $\frac{|A|}{\lambda}$  is an eigen value of  $\text{adj. } A$
- (b) Show that the matrix

$$A = \begin{bmatrix} 1 & 2 & 0 \\ 2 & -1 & 0 \\ 0 & 0 & -1 \end{bmatrix}$$

satisfies its own characteristic equation and hence or otherwise find  $A^{-1}$ .

### Unit-III

6. (a) If  $\log y = \tan^{-1} x$ , show that  $(1+x^2)y_{n+2} + \{2(n+1)x - 1\}y_{n+1} + n(n+1)y_n = 0$  and hence find  $y_3, y_4$  and  $y_5$  at  $x=0$ .
- (b) Show that the radius of curvature  $\rho$  at P on an ellipse  $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$  is given by  $\rho = \frac{CD^3}{ab}$  where CD is the semi-diameter conjugate to CP.
- (c) Expand  $e^x \log(1+y)$  in powers of x and y upto terms of third degree.

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