24002

B. Tech. 1st SemesterF-Scheme Examination, December–2014

MATHEMATICS–I

Paper–MATH–101 F

Time allowed : 3 hours] *[Maximum marks : 100*

Note : ***Question No. 1 is compulsory.*** *Attempt total five questions with selecting* ***one*** *question from each unit. All questions carry equal mark.*

1. (a) What will be the sum and product of Eigen values of identity matrix of order 3×3 and 4 × 4. 2½×8

   (b) Write the formula for the radius of curvature when the curve is given in the implicit and parametric form.

   (c) Find first and second order derivatives from the relation $\log z = x + y + z$

   (d) Write the relationship between Cartesian coordinates and cylindrical polar cords. Also write relationship between Cartesian coordinates and Spherical polar coordinates.

   (e) Define rank matrix. Find rank of matrix

$$\begin{bmatrix} 3 & 1 & 2 & 4 \\ -1 & 0 & 4 & 9 \end{bmatrix}$$

   (f) Define Beta and Gamma function

   (g) Using Cayley Hamilton theorem find A^8 where $A = \begin{pmatrix} 1 & 2 \\ \end{pmatrix}$

Unit–1

2. (a) Test the convergence of the series :

$x + 2x^2 + 3x^3 + 4x^4 + \cdots\cdots\cdots\cdots, \infty$

(b) (i) Show that the series

$$\sum_{n=1}^{\infty} (-1)^{n-1} \sin \frac{1}{n}$$

is conditionally convergent.

(ii) Test the behavior of the series

$$\sum_{n=1}^{\infty} \left(\sqrt[3]{n^3+1} - n\right).$$

3. (a) Test the following series for convergence

$$\frac{x^2}{2\log 2} + \frac{x^3}{3\log 3} + \frac{x^4}{4\log 4} \cdots\cdots \text{to } \infty$$

(b) Prove that the series

$$\frac{\sin x}{1^3} - \frac{\sin 2x}{2^3} + \frac{\sin 3x}{3^3} \cdots\cdots\cdots \text{ converges absolutely.}$$

Unit–2

4. (a) Find the rank of the matrix

$$\begin{pmatrix} 3 & -1 & 2 \\ -6 & 2 & 4 \\ -3 & 1 & 2 \end{pmatrix}$$

by reducing it in its normal form.

(b) Investigate the values of λ and μ so that equations
$2x+3y+5z=9, 7x+3y-2z=8, 2x+3y+\lambda z=\mu$
have (i) no solution
(ii) Unique solution
(iii) More than one solution.

5. (a) Find the eigen values and eigen vectors of the matrix

$$A = \begin{bmatrix} -2 & 2 & -3 \\ 2 & 1 & -6 \\ -1 & -2 & 0 \end{bmatrix}.$$

(b) Reduce the quadratic form
$3x^2 + 5y^2 + 3z^2 - 2yz + 2zx - 2xy$
to the canonical form. Also specify the matrix of transformation.

Unit–3

6. (a) Find the radius of curvature at any point of the curve $r^n = a^n \cos n\theta$

(b) Find the asymptotes of the curve ;
$(x+y)^2 (x+y+2) = x + 9y - 2$

(c) State and prove Euler's theorem on homogeneous functions and use it to find

$$x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} \text{ where u is given by}$$

$$u = \sin^{-1}\left(\frac{x^2+y^2}{\quad}\right)$$

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7. (a) Find the points on the surface $z^2 = xy + 1$, nearest to the origin.

(b) If $\frac{x^2}{a^2+u} + \frac{y^2}{b^2+u} + \frac{z^2}{c^2+u} = 1$ prove that

$$\left(\frac{\partial u}{\partial x}\right)^2 + \left(\frac{\partial u}{\partial y}\right)^2 + \left(\frac{\partial u}{\partial z}\right)^2 = 2\left(x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} + z\frac{\partial u}{\partial z}\right).$$

Unit–4

8. (a) Evaluate by changing the order of integration

$$\int_0^\infty \int_0^x x.e^{-\frac{x^2}{y}}.dy\,dx$$

(b) State and prove the duplication formula

9. (a) Find the volume of sphere $x^2 + y^2 + z^2 = a^2$

(b) Find the area of the portion of the sphere $x^2 + y^2 + z^2 = a^2$ lying inside the cylinder $x^2 + y^2 = ax$.