

24002

**B. Tech. 1st Semester F-Scheme Examination,
December-2014**

MATHEMATICS-I

Paper-MATH-101 F

Time allowed : 3 hours]

[Maximum marks : 100

Note : *Question No. 1 is compulsory. Attempt total five questions with selecting one question from each unit. All questions carry equal mark.*

1. (a) What will be the sum and product of Eigen values of identity matrix of order 3×3 and 4×4 . $2\frac{1}{2} \times 8$
- (b) Write the formula for the radius of curvature when the curve is given in the implicit and parametric form.
- (c) Find first and second order derivatives from the relation $\log z = x + y + z$
- (d) Write the relationship between Cartesian coordinates and cylindrical polar cords. Also write relationship between Cartesian coordinates and Spherical polar coordinates.
- (e) Define rank matrix. Find rank of matrix

$$\begin{bmatrix} 3 & 1 & 2 & 4 \\ -1 & 0 & 4 & 9 \end{bmatrix}$$

- (f) Define Beta and Gamma function
- (g) Using Cayley Hamilton theorem find A^8

where $A = \begin{pmatrix} 1 & 2 \end{pmatrix}$

Unit-1

2. (a) Test the convergence of the series :

$$x + 2x^2 + 3x^3 + 4x^4 + \dots, \infty$$

- (b) (i) Show that the series

$$\sum_{n=1}^{\infty} (-1)^{n-1} \sin \frac{1}{n}$$

is conditionally convergent.

- (ii) Test the behavior of the series

$$\sum_{n=1}^{\infty} (\sqrt[3]{n^3 + 1} - n).$$

3. (a) Test the following series for convergence

$$\frac{x^2}{2 \log 2} + \frac{x^3}{3 \log 3} + \frac{x^4}{4 \log 4} + \dots \text{ to } \infty$$

- (b) Prove that the series

$$\frac{\sin x}{1^3} - \frac{\sin 2x}{2^3} + \frac{\sin 3x}{3^3} - \dots \text{ converges absolutely.}$$

Unit-2

4. (a) Find the rank of the matrix

$$\begin{pmatrix} 3 & -1 & 2 \\ -6 & 2 & 4 \\ -3 & 1 & 2 \end{pmatrix}$$

by reducing it in its normal form.

(3)

24002

- (b) Investigate the values of λ and μ so that equations $2x + 3y + 5z = 9$, $7x + 3y - 2z = 8$, $2x + 3y + \lambda z = \mu$ have
- (i) no solution
 - (ii) Unique solution
 - (iii) More than one solution.

5. (a) Find the eigen values and eigen vectors of the matrix

$$A = \begin{bmatrix} -2 & 2 & -3 \\ 2 & 1 & -6 \\ -1 & -2 & 0 \end{bmatrix}.$$

- (b) Reduce the quadratic form $3x^2 + 5y^2 + 3z^2 - 2yz + 2zx - 2xy$ to the canonical form. Also specify the matrix of transformation.

Unit-3

6. (a) Find the radius of curvature at any point of the curve $r^n = a^n \cos n\theta$
- (b) Find the asymptotes of the curve ;
 $(x + y)^2 (x + y + 2) = x + 9y - 2$
- (c) State and prove Euler's theorem on homogeneous functions and use it to find

$$x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y} \text{ where } u \text{ is given by}$$

$$u = \sin^{-1} \left(\frac{x^2 + y^2}{\dots} \right)$$

7. (a) Find the points on the surface $z^2 = xy + 1$, nearest to the origin.

- (b) If $\frac{x^2}{a^2 + u} + \frac{y^2}{b^2 + u} + \frac{z^2}{c^2 + u} = 1$ prove that

$$\left(\frac{\partial u}{\partial x}\right)^2 + \left(\frac{\partial u}{\partial y}\right)^2 + \left(\frac{\partial u}{\partial z}\right)^2 = 2 \left(x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y} + z \frac{\partial u}{\partial z} \right).$$

Unit-4

8. (a) Evaluate by changing the order of integration

$$\int_0^{\infty} \int_0^x x \cdot e^{-\frac{x^2}{y}} \cdot dy \, dx$$

- (b) State and prove the duplication formula

9. (a) Find the volume of sphere $x^2 + y^2 + z^2 = a^2$

- (b) Find the area of the portion of the sphere $x^2 + y^2 + z^2 = a^2$ lying inside the cylinder $x^2 + y^2 = ax$.