

7. (a) If $u = \tan^{-1} \left(\frac{y}{x} \right)$ prove that

$$x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y} = \sin 2u$$

and

$$x^2 \frac{\partial^2 u}{\partial x^2} + y^2 \frac{\partial^2 u}{\partial y^2} + 2xy \frac{\partial^2 u}{\partial x \partial y} = 2 \cos 3u \sin u.$$

- (b) A rectangular box open at the top is to have a volume of 32 c.c. Find the dimensions of the box requiring least material for its construction.

IV

8. (a) Find the area of the region bounded by the parabola $y = 4x - x^2$ and the line $y = x$.
 (b) Find, by double integration, the volume of the solid

generated by revolving the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ about the y-axis.

9. (a) Find the volume of the ellipsoid

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$$

- (b) Prove that

$$\beta(m, n) = \frac{\Gamma m \Gamma n}{\Gamma m+n}, \text{ hence evaluate } \Gamma 1/2.$$

**B.Tech. 1st Semester F-Scheme Examination,
 December-2016
 MATHEMATICS-I
 Paper-MATH-101-F**

Time allowed : 3 hours] [Maximum marks : 100

Note : Question No. 1 is compulsory. Attempt total five questions with selecting one question from each unit. All questions carry equal marks.

1. (a) Discuss the behavior of series

$$\frac{\sqrt{2}-1}{3^3-1} + \frac{\sqrt{3}-1}{4^3-1} + \frac{\sqrt{4}-1}{5^3-1} + \frac{\sqrt{5}-1}{6^3-1} + \dots$$

- (b) Prove that

$$\log(1+x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \dots$$

- (c) Find asymptotes parallel to co-ordinate axis of the curve

$$x^2 y^2 - x^2 y - xy^2 + x + y + 1 = 0$$

- (d) Define rank of matrix. Find rank of matrix

$$\begin{bmatrix} 1 & 2 & 3 & 4 \\ -2 & 0 & 5 & 7 \end{bmatrix}$$

- (e) Using Cayley-Hamilton theorem, find A^6 if

$$\begin{bmatrix} 2 & 1 \\ 5 & -2 \end{bmatrix}$$

- (f) Write the relation between Cartesian coordinates and cylindrical coordinates. Also write the relationship between Cartesian coordinates and Spherical polar coordinates.
- (g) Define Beta and Gamma functions.
- (h) State Leibnitz theorem for the nth derivative of the product of two functions.

U

2. (a) Examine the convergence of the series whose nth term is $\sqrt{n^4 - 1}$.
- (b) Discuss the convergence of the series $\frac{2}{1}x + \frac{9}{8}x^2 + \frac{64}{81}x^3 + \dots$ to ∞ .

3. (a) Test for absolute/conditionally convergence of the series $2 - \frac{4}{2!} + \frac{8}{3!} - \frac{32}{5!} + \dots$.
- (b) For what values of x are the following series convergent: $\frac{1}{1-x} + \frac{1}{(1-x)^2} + \frac{1}{3(1-x)^3} + \dots \infty$.

U

4. (a) Find non-singular matrices P and Q such that PAQ is in normal form for the matrix

$$A = \begin{bmatrix} 0 & 2 & 4 \\ 2 & 3 & 4 \\ 4 & 8 & 12 \end{bmatrix}$$

- (b) Find the value of λ such that the following equations have unique solution:
 $\lambda x + 2y - 2z - 1 = 0$, $4x + 2\lambda y - z - 2 = 0$,
 $6x + 6y + \lambda z - 3 = 0$
 and use matrix method to solve these equations when $\lambda = 2$.

5. (a) If λ is an eigen value of a non-singular matrix A, show that
 (i) $\lambda - 1$ is an eigen value of $A - I$
 (ii) $\frac{|A|}{\lambda}$ is an eigen value of $\text{adj. } A$
- (b) Show that the matrix

$$A = \begin{bmatrix} 1 & 2 & 0 \\ 2 & -1 & 0 \\ 0 & 0 & -1 \end{bmatrix}$$

satisfies its own characteristic equation and hence or otherwise find A^{-1} .

Unit-III

6. (a) If $\log y = \tan^{-1} x$, show that $(1+x^2)y_{n+2} + \{2(n+1)x - 1\}y_{n+1} + n(n+1)y_n = 0$ and hence find y_3, y_4 and y_5 at $x=0$.
- (b) Show that the radius of curvature ρ at P on an ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ is given by $\rho = \frac{CD^3}{ab}$ where CD is the semi-diameter conjugate to CP.
- (c) Expand $e^x \log(1+y)$ in powers of x and y upto terms of third degree.