

94059

B. Sc. (Hons.) Mathematics

5th Semester

Examination – March, 2021

REAL ANALYSIS

Paper : BHM-351

Time : Three Hours]

[Maximum Marks : 60

Before answering the questions, candidates should ensure that they have been supplied the correct and complete question paper. No complaint in this regard, will be entertained after examination.

Note : Attempt five questions in all, selecting one question from each Section. Question No. 9 is compulsory.

All questions carry equal marks.

SECTION - I

- 1/ (a) If a function f is defined on $[0, b]$, $b > 0$ by $f(x) = x^3$, then show that f is Riemann integrable on $[0, b]$ and $\int_0^b f dx = \frac{b^4}{4}$.

- (b) Prove that every bounded monotonic function is a integrable function.

2. (a) If a function f is continuous on $[a, b]$ and $g(x) = \int_a^x f(t) dt$, then prove that g is differentiable on $[a, b]$ and $g' = f$.

- (b) Prove that $\frac{1}{\pi} \leq \int_0^1 \frac{\sin \pi x}{1+x^2} dx \leq \frac{2}{\pi}$.

SECTION - II

3. (a) Show that the integral $\int_0^{\pi/2} \frac{\sin^m x}{x^n} dx$ exists if and only if $n < m + 1$.

- (b) Examine the convergence of the improper integral $\int_0^{2a} \frac{1}{(x-a)^2} dx$.

4. (a) Evaluate $\int_0^{\pi} \frac{\log(1 + \alpha \cos x)}{\cos x} dx$ for $|\alpha| < 1$.

- (b) If $\int_a^t f dx$ is bounded for all $t \geq a$ and g is a bounded and monotonic function for $x \geq a$ tending to '0' as $x \rightarrow \infty$, then $\int_a^{\infty} fg dx$ is convergent at ∞ .

SECTION - III

5. (a) Let (X, d) be a metric space. Prove that a subset of X is closed iff its complement is open.
- (b) Prove that in a metric space (X, d) , the union of arbitrary collection of open sets is open.
6. (a) If A and B are subsets of metric space (X, d) , then prove that $\overline{A \cup B} = \overline{A} \cup \overline{B}$.
- (b) Prove that every complete metric space is of the second category as a subset of itself.

SECTION - IV

7. (a) Let f be a function of (X, d) into (Y, d^*) . Then f is continuous if and only if $f^{-1}(G)$ is open in X whenever G is open in Y .
- (b) Prove that a compact subset of a metric space is closed and bounded.
8. (a) Prove that a continuous image of a connected space is connected.
- (b) Prove that a metric space (X, d) is compact if and only if it has Bolzano Weierstrass property.

SECTION - V

9. (a) Define norm of a partition of P .
- (b) State Abel's test for convergence of improper integral.
- (c) Define uniform continuity of a function.
- (d) Discuss the convergence of $\int_0^1 \frac{dx}{x^{1/3}(1+x^2)}$.
- (e) Define lower and upper sums.
- (f) State Baire's category Theorem.