Roll No. ....

94062

B. Sc. (Hons.) Mathematics
5th Semester Old/New Scheme
Examination – February, 2022

INTEGRAL EQUATION

Paper : BHM-354

Time : Three Hours] *[Maximum Marks : 60*

Before answering the questions, candidates should ensure that they have been supplied the correct and complete question paper. No complaint in this regard, will be entertained after examination.

Note : Attempt *five* questions in all, selecting *one* question from each Section. Question No. 9 (Section-V) is *compulsory*. All questions carry equal marks.

SECTION – I

1. (a) Using the method of successive approximation solve the integral equation : 6

$$u(x) = x - \int_0^x (x-\xi)\,u(\xi)\,d\xi.$$

(b) Solve the integral equation $\sin x = \lambda \int_0^x e^{x-\xi} u(\xi)\,d\xi$. 6

2. (a) By the Resolvent Kernel method, solve Volterra integral equation of second kind : 6

$$u(x) = f(x) + \lambda \int_0^x e^{x-\xi} u(\xi)\,d\xi.$$

(b) Reduce the initial value problem $y'' + xy = 1$, $y'(0) = 0 = y(0)$ to Volterra integral equation. 6

SECTION – II

3. (a) Reduce the boundary value problem, 6

$y'' + A(x)y' + B(x)y = g(x)$, $a \le x \le b$, $y(a) = c_1$, $y(b) = c_2$ to a Fredholm integral equation.

(b) Find the first two approximation of the solution of Fredholm integral equation : 6

$$u(x) = 1 + \int_0^1 K(x,\xi)\,u(\xi)\,d\xi \text{ where } K(x,\xi) = \begin{bmatrix} x & 0 \le x \le \xi \\ \xi & \xi \le x \le 1 \end{bmatrix}.$$

4. (a) Solve the following integral equation of second kind by the method of successive approximations to third order : 6

$$\phi(x) = 1 + \lambda \int_0^1 (x+\xi)\,\phi(\xi)\,d\xi, \phi_0(x) = 1.$$

(b) Find the resolvent Kernels for the Kernel $K(x,\xi) = e^{(x+\xi)}$, $a = 0$, $b = 1$. 6

SECTION – III

5. (a) Construct Green's function for the equation $x\frac{d^2u}{dx^2}+\frac{du}{dx}=0$ with the conditions $u(x)$ is bounded as $x \to 0$, $u(1)\ \mu u'(1)$, $\mu \neq 0$. 6

(b) Reduce the Bessel's differential equation $x^2\frac{d^2u}{dx^2}+x\frac{du}{dx}+(\lambda x^2-1)u=0$ with the conditions $u(0) = 0$, $u(1) = 0$ into an integral equation, using Green's function. 6

6. (a) Reduce the boundary value problem $\frac{d^2u}{dx^2}+\frac{\pi^2}{4}u=\lambda u+\cos\frac{\pi x}{2}, u(-1)=u(1)$ and $u'(-1)=u'(1)$ to an integral equation, using Green's function. 6

(b) Discus construction of Green's function by variation of parameter method. 6

SECTION – IV

7. For the Fredholm integral equation $y(x)=\lambda\int_a^b K(x,\xi)\, y(\xi)d\xi$ with symmetric Kernel, prove that : 12

(i) The eigen functions corresponding to two different eigen values are orthogonal over (a, b).

(ii) The eigen values are real.

8. (a) Determine the eigen values and eigen functions for the following integral equations : 6

$$\phi(x)=\lambda\int_0^1(5x\xi^3+4x^2\xi)\phi(\xi)d\xi.$$

(b) Solve the following integral equation by the method of Fredholm Resolvent Kernal : 6

$$\phi(x)=(\sin x-\frac{x}{4})+\frac{1}{4}\int_0^{\pi/2}x\xi\,\phi(\xi)d\xi.$$

SECTION – V

9. (a) What is Fredholm integral equation of first kind and given an example ? 12

(b) Show that the function $y(x)=(1+x^2)^{-\frac{3}{2}}$ is solution of the Volterra integral equation $y(x)=\frac{1}{1+x^2}-\int_0^x\frac{\xi}{1+x^2}y(\xi)d\xi$.

(c) What do you mean by initial value problem ?

(d) Write four basic properties of Green's function.

(e) What do you mean by homogenous integral equation of Fredholm type ?

(f) Define Iterated Kernel and Neumann series for Fredholm equation.