Roll No. .....

94062

B. Sc. (Mathematics) (Hons.)

5th Semester

Examination – March, 2021

INTEGRAL EQUATION

Paper : BHM-354

Time : Three Hours] *[Maximum Marks : 60*

Before answering the questions, candidates should ensure that they have been supplied the correct and complete question paper. No complaint in this regard, will be entertained after examination.

Note : Attempt *five* questions in all, selecting *one* question from each Section. Question No. **9** is ***compulsory***. All questions carry equal marks.

SECTION– I

1. (a) Convert the differential equation :

$$y''(x) - 3y'(x) + 2y(x) = 4\sin x$$

with the initial conditions $y(0) = 1, y'(0) = -2$ into Volterra's integral equation of second kind.

(b) Find the resolvent kernels of the Volterra's integral equation with the kernel $K(x, \xi) = \begin{cases} e^{x-\xi} & x > \xi \\ 0 & \xi > x \end{cases}$

2. (a) Solve the Volterra's integral equation :

$$\phi(x) = (1 - 2x - 4x^2) + \int_0^x \left[3 + 6(x - \xi) - 4(x - \xi)^2\right] \phi(\xi)\, d\xi$$

(b) Solve the Abel integral equation :

$$f(x) = \int_0^x \frac{\phi(\xi)}{(x - \xi)^{\alpha}}\, d\xi \text{ where, } 0 < \alpha < 1$$

SECTION– II

3. (a) Obtain Fredholm integral equation of second kind corresponding to the boundary value problem :

$$\frac{d^2\phi}{dx^2} = 1 - x\phi(x) \text{ with } \phi(0) = 0 \text{ and } \phi(1) = 1$$

(b) Find the iterated kernals for the following kernels :

(i) $K(x, \xi) = x - \xi$, if $a = 0, b = 1$

(ii) $K(x, \xi) = e^x \cos\xi, a = 0, b = \pi$

4. (a) Using Fredholm determinants, find the resolvent kernel of $k(x, \xi) = \sin x - \sin \xi$; $0 \le x \le 2\pi$, $0 \le \xi \le 2\pi$

(b) Solve the following integral equation :

$$\phi(x) = \frac{6}{5}(1 - 4x) + \lambda \int_0^1 (x \log \xi - \xi \log x)\, \phi(\xi)\, d\xi$$

SECTION– III

5. (a) Find the Green's functions for given B.V.P.

$$\frac{d^2y}{dx^2} + y = 0, y(0) = y(1), y'(0) = y'(1)$$

(b) Reduce the B.V.P. $\frac{d^2\mu}{dx^2} + \frac{\pi^2}{4}\mu = \lambda\mu + \cos\frac{\pi x}{2}$

$\mu(-1) = \mu(1)$ and $u'(-1) = \mu'(1)$ to an I.E.

6. (a) Explain the method of variation of parameters to construct the Green's function.

(b) Using Green's function, solve $\frac{d^2y}{dx^2} - y = x$ with

$y(0) = y(1) = 0$

SECTION– IV

7. (a) Find the eigen values and eigen function of the homogenous equation :

$$\phi(x) = \lambda \int_0^{\pi} k(x, \xi)\, \varphi(\xi)\, d\xi$$

Where $k(x, \xi) = \begin{cases} \cos x \sin \xi & 0 \le x \le \xi \\ \cos \xi \sin x & \xi \le x \le \pi \end{cases}$

(b) If $k(x,\xi)$ is symmetric and $\phi_m(x)$, $\phi_n(x)$ are fundamental functions of $K(x, \xi)$ for λ_m and λ_n respectively $(\lambda_m \neq \lambda_n)$, then show that :

$$\int_a^b \phi_m(x)\, \phi_n(x)\, dx = 0$$

8. (a) Solve the symmetric integral equation :

$$\phi(x) = F(x) + \lambda \int_a^b k(x, \xi)\, \phi(\xi)\, d\xi$$

where $k(x, \xi) = f(x)\, f(\xi)$

(b) Solve the symmetric integral equation :

$$\phi(x) = 1 + \lambda \int_0^{\pi} \cos(x + \xi)\, \phi(\xi)\, d\xi$$

SECTION– V

9. (a) Define Volterra integral equation of second kind.

(b) Show that the function $\phi(x) = e^x[2x - (2/3)]$ is a solution of Fredholm integral equation :

$$\phi(x) - 2\int_0^1 e^{x-\xi}\, \phi(\xi) d\xi = 2xe^x$$

(c) Write a short note on Alternative Fredholm.

(d) Write Fredholm's first series.

(e) Define Separable kernel with *two* examples.

(f) State Hadamard's Theorem.
