

94060

**B. Sc. (Hons.) Mathematics
5th Semester Old/New Scheme
Examination – February, 2022**

GROUPS AND RINGS

Paper : BHM-352

Time : Three Hours]

[Maximum Marks : 60

Before answering the questions, candidates should ensure that they have been supplied the correct and complete question paper. No complaint in this regard, will be entertained after examination.

Note : Attempt five questions in all, selecting one question from each Section. Question No. 9 (Section – V) is compulsory. All questions carry equal marks.

SECTION – I

1. (a) Prove that a necessary and sufficient condition for a non-empty finite subset H of group (G, \cdot) to be a subgroup is that H must be closed with respect to multiplication. 6

- (b) State and prove Lagrange's theorem for finite groups. 6

2. (a) If G is the additive group of integers and H is the subgroup of G obtained on multiplying the elements of G by 4, find the index of H in G . 6
- (b) Prove H is a normal subgroup of G iff $xHx^{-1} = H$ for all $x \in G$. 6

SECTION – II

3. (a) State and prove Fundamental theorem on Homomorphism of Groups. 6
- (b) If G is a finite abelian group of order n and m is a positive integer such that $(m, n) = 1$ then show that $f : G \rightarrow G$ defined by $f(x) = x^m$ is an automorphism. 6
4. (a) Let $Z(G)$ be the centre of a group G . If G/Z is cyclic, then prove that G is abelian. 6
- (b) If $S = \{1, 2, 3, 4, 5, 6, 7\}$ and $f = (1, 3), g = (2, 4, 6)$, show that $f \circ g = g \circ f$. 6

SECTION – III

5. (a) Show that centre of a ring R is a subring of R . 6

- (b) Prove that every field is a principal ideal ring. 6
6. (a) Prove that every homomorphic image of a ring R is isomorphic to some quotient ring. 6
- (b) An ideal S of a commutative ring R with unity is maximal iff R/S is a field. 6

SECTION - IV

7. (a) The ring of Gaussian integers is an Euclidean domain. <https://www.mdustudy.com> 6
- (b) Show that every non-zero prime ideal of a principal ideal domain is maximal. 6
8. (a) If F is a field, then $F[x]$ is a principal ideal ring. 6
- (b) Show that if a is an irreducible element of a unique factorization domain R then a must be prime. 6

SECTION - V

(Compulsory Question)

9. (a) Find the principal ideal generated by 3, in the ring of integers. 12
- (b) Show that the mapping $f : C \rightarrow C$ defined by $f(a + ib) = a - ib$ is a homomorphism.

- (c) Prove that the product of two units $a, b \in R$ is also a unit of R .
- (d) Let $G = \{1, 2, 3, 4\}$ be the group w.r.t. multiplication modulo 5. Find order of each element.
- (e) Prove that every group of prime order is cyclic.
- (f) Prove that identity mapping is the only inner automorphism for an abelian group.