

Roll No. ....

**94080**

**B. Sc Mathematics (Hons.)**

**5th Semester**

**Examination – March, 2021**

**GROUPS & RINGS**

**Paper :BHM-352**

**Time : Three Hours ]**

**[ Maximum Marks : 60**

*Before answering the questions, candidates should ensure that they have been supplied the correct and complete question paper. No complaint in this regard, will be entertained after examination.*

**Note :** Attempt five questions in all, selecting *one* question from each Section. Question No. 9 (Section - V) is *compulsory*. All questions carry equal marks.

**SECTION – I**

1. (a) Show that union of two subgroups is a subgroup if and only if one is contained in other. 6

- (b) Show that every subgroup of a Cyclic group is cyclic. 6

2. (a) Show that the order of every element of a finite group is a divisor of the order of the group. 6

- (b) If H is the only subgroup of finite order in the group G, then prove that H is the normal subgroup of G. 6

**SECTION – II**

3. (a) Show that every homomorphic image of a group G is isomorphic to some quotient group of G. 6

- (b) If a is a fixed element of a group G then show that the mapping  $T_a : G \rightarrow G$  defined as  $T_a(x) = a^{-1}xa$  in an automorphism of G. 6

4. (a) If p is a prime number and G is non-abelian group of order  $p^3$  then show that Z(G) has exactly p elements. 6

- (b) Show that every group is isomorphic to a permutation group. 6

### SECTION - III

5. (a) Show that the set  $(\{0, 1, 2, 3, 4\}, +_5, \times_5)$  is a field. 6

(b) If  $R$  is a Commutative ring with unity and has no proper ideals then show that  $R$  is a field. 6

6. (a) If  $S$  and  $T$  are two ideals of a ring  $R$ , then show that  $(S+T)/S \cong T/(S \cap T)$ . 6

(b) If an ideal  $S$  of a commutative ring  $R$  with unity is maximal then show that  $R/S$  is a field. 6

### SECTION - IV

7. (a) If  $R$  is an Euclidean ring and  $S$  be an ideal of  $R$  then show that there exist an element  $a_0 \in S$  such that  $S = \{a_0 r : r \in R\}$ . 6

(b) Show that every non-zero prime ideal of a principal ideal domain is maximal. 6

8. (a) Show that the polynomial ring  $Z[x]$  over the ring of integer is not a principal ideal ring. 6

(b) Show that the polynomial  $x^4 + 1$  is irreducible over  $Q$ . 6

### SECTION - V

9. (a) If  $G = \{0, 1, 2, 3, 4, 5\}$  is a group under the binary operation addition modulo 6, find the order of elements 2 and 4. 2

(b) How many generators are there of the cyclic group  $G$  of order 8? 2

(c) Use Lagrange's theorem to show that any group of prime order can have no proper subgroup. 2

(d) Write  $\begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 6 & 5 & 4 & 3 & 1 & 2 \end{pmatrix}$  as the product of disjoint cycles. 2

(e) If  $S$  is an ideal of a ring  $R$  with unity such that  $1 \in S$  then show that  $S = R$ . 2

(f) Show that every homomorphic image of a commutative ring is commutative. 2