Roll No. ....

41182

B. Sc. (Pass Course) 4th Semester

Examination – May, 2019

MATHEMATICS - II (SPECIAL FUNCTIONS AND INTEGRAL TRANSFORMS)

Paper : 12BSM242

Time : Three hours] *[Maximum Marks : 40*

Before answering the questions, candidates should ensure that they have been supplied the correct and complete question paper. No complaint in this regard, will be entertained after examination.

Note : Attempt ***five*** questions in all, selecting ***one*** question from each Section. Question No. 9 (Section – V) is ***compulsory***.

SECTION – I

1. (a) Find the series solution of the following differential equation about $x = 0$: $3\frac{1}{2}$

$$x(1-x)\frac{d^2y}{dx^2} - 3x\frac{dy}{dx} - y = 0$$

(b) Find power series solution of following initial value problem : $3\frac{1}{2}$

P. T. O.

$$(x^2-1)\frac{d^2y}{dx^2} + 3x\frac{dy}{dx} + xy = 0,\ y(0) = 2,\ y'(0) = 3$$

2. (a) Solve the following equation in terms of Bessel's function : $3\frac{1}{2}$

$$x^2\frac{d^2y}{dx^2} + x\frac{dy}{dx} + (x^2 - 25)\,y = 0$$

(b) State and prove orthogonality relation of Bessel's function. $3\frac{1}{2}$

SECTION – II

3. (a) Using Rodrigue's Formula, show that $P_n(x)$ satisfies the differential equation $\frac{d}{dx}\left[(1-x^2)\frac{d}{dx}P_n(x)\right] + n(n+1)P_n(x) = 0$ Where $P_n(x)$ is Legendre polynomial of order n. $3\frac{1}{2}$

(b) Discuss orthogonality of Legendre's polynomial.

4. (a) Expand e^{2x} in a series of Hermite's polynomial. $3\frac{1}{2}$

(b) If $\phi_n(x) = e^{-\frac{x^2}{2}}H_n(x)$, where $H_n(x)$ is a Hermite's polynomial of degree n, then show that : $3\frac{1}{2}$

$$\int_{-\infty}^{\infty}\phi_{m,}(x)\,\phi_n(x)\,dx = 2^2 \times n!\sqrt{\pi}\,\delta_{mn}$$

where δ_{mn} is Kronecker delta.

SECTION – III

5. (a) Evaluate $\int_0^\infty t e^{-t} \sin^4 t \, dt$ using Lap

transform.

(b) Find inverse Laplace transform of $\operatorname{Tan}^{-1}\left(\frac{2}{s^2}\right)$

6. (a) Using convolution theorem, eval

$L^{-1}\left(\frac{1}{(s-1)(s+3)}\right)$.

(b) Solve the following differential equa

$t\frac{d^2y}{dx^2} + (t-1)\frac{dy}{dt} - y = 0, y(0) = 5, y(\infty) = 0$ us

Laplace transform method.

SECTION – IV

7. (a) Find Fourier transform of function :

$$f(x) = \begin{cases} xe^{-x} & , \quad x > 0 \\ -0 & , \quad x < 0 \end{cases}$$

(b) Find finite Fourier cosine transform of :

$$f(x) = \begin{cases} 1 & , \quad 0 < x < \frac{\pi}{2} \\ -1 & , \quad \frac{\pi}{2} < x < \pi \end{cases}$$

8. (a) The temperature u in a semi - infinite rod is determined by $\frac{\partial u}{\partial t} = c^2 \frac{\partial^2 u}{\partial x^2}; 0 \le x < \infty$ with conditions : $3\frac{1}{2}$

(i) u = 0 when t = 0, x > 0

(ii) $\frac{\partial u}{\partial x} = -\mu$ when $x = 0$

(iii) $\frac{\partial u}{\partial x} \to 0$ as $x \to \infty$

Determine temperature formula.

(b) Find finite cosine transform of $\left(1 - \frac{x}{\pi}\right)^2$. $3\frac{1}{2}$

SECTION – V

9. (a) Find radius of convergence of series $\sum_{m=0}^{\infty} m!\, x^m$ 2

(b) Define relation between Fourier and Laplace transform. 2

(c) Define Hermite's differential equation. 2

(d) Prove that $P_n(1) = 1$ where P_n is Legendre polynomial of degree n. 2

(e) Find finite Fourier sine transform of $f(x) = x^3$. 2

(f) Give first shifting property of inverse Laplace Transform. 2