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B. Sc. (Pass Course) 4th Semester Examination – May, 2019

MATHEMATICS - I (SEQUENCES AND SERIES)

Paper : 12BSM241

Time : Three hours / Maximum Marks : 40

Before answering the questions, candidates should ensure that they have been supplied the correct and complete question paper. No complaint in this regard, will be entertained after examination.

Note : Attempt five questions in all, selecting at least one question from each Section. Section V is compulsory.

SECTION – I

- 1. (a) Prove that set of rationals is not order complete. 7
(b) If x and y are two positive real numbers, then there exists a natural number n such that nx > y. 7
2. (a) Prove that every set satisfying Heine Borel property is a compact set. 7
(b) The derived set of any set is a closed set. Prove it.

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SECTION – II

- 3. (a) Let <an> be a sequence s. t. an ≠ 0 for all n ∈ N and (an+1)/an → l. If |l| < 1 then lim n→∞ an = 0.

(b) Show that :

lim n→∞ [1/n^2 + 1/(n+1)^2 + + 1/(2n)^2] = 0

- 4. (a) Test the convergence of the series 1/(1+x) + x/(1+x^2) + x^2/(1+x^3) + where x > 0.

- (b) If the series sum n=1 to ∞ an converges, then lim n→∞ an = 0. Is the converse true? If not show by an example?

SECTION – III

- 5. (a) If sum n=1 to ∞ an = 1/2 + 1/3^2 + 1/2^3 + 1/3^4 + then show that cauchy root test establishes the convergence of the series. sum n=1 to ∞ an while's Alembert's ratio test fails to do so.

(b) Test convergence of series :

$$1 + \frac{x}{2} + \frac{21}{3^2}x^2 + \frac{31}{4^3}x^3 + \frac{41}{5^4}x^4 \dots (x > 0).$$

6. (a) Discuss convergence of series $\sum_{n=1}^{\infty} \frac{(n!)^2}{2n!}$

where $x > 0$.

(b) Using Cauchy's condensation test, discuss convergence of $\sum_{n=2}^{\infty} \frac{\log n}{n}$.

SECTION - IV

7. (a) Prove that if $\sum_{n=1}^{\infty} a_n$ is conditionally convergent then the series of its positive and the series of negative terms are both divergent.

(b) Test the convergence of the series :

$$\sum_{n=3}^{\infty} \frac{(n^3 + 1)^{1/3} - n}{\log n}$$

8. (a) Discuss the convergence of infinite product :

$$\prod_{n=1}^{\infty} \left(1 + \frac{x}{n}\right), x \neq 0$$

(b) Show that the series $\left(1 - \frac{2}{3}\right) + \left(1 - \frac{8}{9}\right) + \left(1 - \frac{26}{27}\right) + \dots$ is convergent but when parenthesis are removed, it oscillates finitely.

SECTION - V

9. (a) Give examples to show that supremum of a set may or may not belong to the set. 2

(b) Give example of a set which has three limit points. 2

(c) Define absolute convergence and conditional convergence of $\sum_{n=1}^{\infty} a_n$. 2

(d) Give an example of a sequence which is bounded but not monotonic. 2

(e) Show that the series : 2

$$1^2 + 2^2 + 3^2 + \dots \text{ diverges to } +\infty$$

(f) Show that infinite product $\prod_{n=1}^{\infty} \left(1 + \frac{1}{n^2}\right)$ is convergent. 2