

Roll No. ....

21181

B. Sc. (Pass Course) 2nd Semester  
Examination – May, 2019

MATHEMATICS - I (NUMBER THEORY &  
TRIGONOMETRY)

Paper : 12BSM 121

Time : Three hours ] [ Maximum Marks : 40

Before answering the questions, candidates should ensure that they have been supplied the correct and complete question paper. No complaint in this regard, will be entertained after examination.

Note : Attempt any five questions in all, selecting one question from each Section. Section - V is compulsory.

SECTION - I

- 1. (a) Find the g. c. d of 858 and 325 and express it in the form  $m \cdot 858 + n \cdot 325$ . 3 1/2

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21181

- (b) Find the remainder if  $3^{40}$  divided by 23. 3 1/2

- 2. (a) State and prove Fermat's theorem. 3 1/2

- (b) Find the general solution in positive integers of  $13x - 17y = 5$ . 3 1/2

SECTION - II

- 3. (a) Euler's function  $(\phi)$  is a multiplicative function. Prove it. 3 1/2

- (b) Prove that  $2, 4, 6, \dots, 2m$  is a CRS (mod  $m$ ) if  $m$  is odd Where CRS is complete residue system. 3 1/2

- 4. (a) If  $p$  and  $q$  are distinct odd primes, then, 
$$\left(\frac{p}{q}\right)\left(\frac{q}{p}\right) = (-1)^{\frac{p-1}{2} \cdot \frac{q-1}{2}}$$
 3 1/2

(b) Show that  $d(n) = d(n+1) = d(n+2) = d(n+3)$

$$n = 4503.$$

**SECTION - III**

5. (a) Show that there are  $q$  and only  $q$  distinct values of  $(\cos \theta + i \sin \theta)^q$ ,  $q$  being any the integer.

(b) Show that :

$$\tan 7\theta = \frac{7 \tan \theta - 35 \tan^3 \theta + 21 \tan^5 \theta - \tan^7 \theta}{1 - 21 \tan^2 \theta + 35 \tan^4 \theta - 7 \tan^6 \theta}$$

6. (a) Prove that :

$$\sin(\alpha + i\beta) = \frac{1}{2}(e^{-\beta} + e^{\beta})\sin \alpha - \frac{i}{2}(e^{-\beta} - e^{\beta})\cos \alpha$$

(b) If  $\tan(A + iB) = x + iy$ , prove that :

$$x^2 + y^2 + 2x \cot 2A = 1$$

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**SECTION - IV**

7. (a) Find the general and principal value of  $\log(-1+i) - \text{Log}(-1-i)$ . 3  $\frac{1}{2}$

(b) Solve the equation : 3  $\frac{1}{2}$

$$\tan^{-1} \frac{1}{4} + 2 \tan^{-1} \frac{1}{5} + \tan^{-1} \frac{1}{6} + \tan^{-1} \frac{1}{x} = \frac{\pi}{4}$$

8. (a) Find the sum of sines of  $n$  angles when the angles are in arithmetical progression. 3  $\frac{1}{2}$

(b) Sum the series : 3  $\frac{1}{2}$

$$1 + x \cos \theta + x^2 \cos 2\theta + \dots + x^{n-1} \cos(n-1)\theta, |x| < 1$$

**SECTION - V**

9. (a) Solve for  $x$  : 6  $\times$  2 = 2

$$\tan^{-1} 2x + \tan^{-1} 3x = \frac{\pi}{4}$$

(b) Find the general value of  $\text{Log}(-5)$ .

(c) State Gauss lemma.

(d) State Demoivre's theorem.

(e) Find  $\phi(n)$  when  $n = 68$ .

(f) Evaluate  $\mu(187)$ .

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