11181

B. Sc. (Pass Course) 1st Semester (Regular/Re-appear/Improvement) (Mercy Chance) Examination – December, 2023

MATHEMATICS – I (Algebra)

Paper : 12BSM111

Time : Three Hours] *[Maximum Marks : 40*

Before answering the questions, candidates should ensure that they have been supplied the correct and complete question paper. No complaint in this regard, will be entertained after examination.

Note : Attempt ***five*** questions in all, selecting ***one*** question from each Section. Question No. 9 (Section – V) is ***compulsory***. Marks are indicated against each question.

SECTION – I

1. (a) Every square matrix A can be expressed in one and only one way as $P+iQ$, where P and Q are Hermitian matrices. 3.5

(b) Reduce the matrices $\begin{bmatrix} 1 & 2 & 3 \\ 1 & 4 & 2 \\ 2 & 6 & 5 \end{bmatrix}$ to the row echelon form. Also find the rank. 3.5

P. T. O.

2. (a) Find non-singular matrices P and A such that PAQ is in normal form where $A = \begin{bmatrix} 3 & 1 & 2 & 1 \\ 1 & 4 & 6 & 1 \\ 2 & -3 & 1 & -2 \end{bmatrix}$. 3.5

(b) Find the characteristic vectors of the matrix : 3.5

$$\begin{bmatrix} -2 & 2 & -3 \\ 2 & 1 & -6 \\ -1 & -2 & 0 \end{bmatrix}$$

SECTION – II

3. (a) For what values of a and b the system of equations $x+y+5z=0,\ x+2y+3az=b, x+3y+az=1$ have

(i) no solution

(ii) unique solution

(iii) infinitely many solution. 3.5

(b) If A is Skew-Hermitian and (A–I) is non-singular, show that $P=(A-I)(A-I)^{-1}$ is unitary. 3.5

4. (a) Reduce the bilinear form :

$$x_1y_1 + x_1y_3 - x_2y_1 + x_2y_2 + x_3y_3$$

to the canonical form. Also find the equations of transformations. 3.5

(b) Prove that $9x^2 + y^2 + 4z^2 + 6xy - 12xz - 4yz$ is positive semi-definite. 3.5

SECTION – III

5. (a) Solve the equation $4x^4 + 8x^3 + 13x^2 + 2x + 3 = 0$, it being given that the sum of two roots is zero. 3.5

(b) If the product of two roots of the equation $x^4 + px^3 + qx^2 + rx + s = 0$, be equal in magnitude but opposite in sign to the product of the other two, show that $p^2s + r^2 = 4qs$. 3.5

6. (a) If α, β, γ are the roots of the equation $x^3 + px^2 + qx + r = 0$, find the equation whose roots are $\alpha^2 - \beta\gamma, \beta^2 - \gamma\alpha, \gamma^2 - \alpha\beta$. 3.5

(b) Find the equation of squared differences of the roots of cubic $x^3 + 6x^2 + 2 = 0$. 3.5

SECTION – IV

7. (a) Solve the equation $x^3 - 3x^2 + 12x + 16 = 0$ by Cardon's method. 3.5

(b) Solve by method of resolution into quadratic factors $x^4 - 2x^3 - 5x^2 + 10x - 3 = 0$. 3.5

8. (a) Solve the equation $x^4 - 4x^3 - 4x^2 - 24x + 15 = 0$ by Ferrari's method. 3.5

(b) Show that for all values of c, the equation $x^5 + 5x^2 + 3x + c = 0$ has at least two imaginary roots. 3.5

SECTION – V

9. (a) Define characteristic roots. 2

(b) Prove that invers of an orthogonal matrix is ± 1. 2

(c) If A is non singular, prove that the eigen values of A^{-1} are the reciprocals of the eigen values of A. 2

(d) State fundamental theorem of Algebra. 2

(e) Form an equation with rational coefficients, two of whose roots are $1 + 5i$, $5 - i$. 2

(f) If A and B are Hermitian, show that AB is Hermitian iff $AB = BA$. 2