Roll No. .....

97663

B.C.A. 1st Semester (New) Examination–December, 2013

Mathematics

Paper-BCA-103

Time : 3 hours *Max. Marks : 80*

Before answering the questions, candidates should ensure that they have been supplied the correct and complete question paper. No complaint in this regard will be entertained after the examination.

Note : Attempt **five** questions in all, selecting **one** question from each unit. Question no. **1** is **compulsory**.

1. (a) Define union and Intersection of set with examples. *2×8=16*

(b) Write the set in Roster form $A = \{x : |x-3| < 8, x \text{ is an integer}\}$

(c) Construct 2×2 Matrix $A = [a_{ij}]$, such that $a_{ij} = \frac{(i+2j)^2}{2}$

(d) Define equivalence relation with example

(e) Evaluate $\lim_{x \to 0}\left(\frac{e^x - e^{-x}}{x}\right)$

(f) if $y = \frac{1-\sqrt{x}}{1+\sqrt{x}}$, find $\frac{dy}{dx}$

(g) $\int \frac{x}{1+\sqrt{x}}\,dx$

(h) $\int_0^{\pi/2} \frac{\sin x}{\cos x + \sin x}\,dx$

Unit-I

2. (a) Prove $(A \cup B)' = A' \cap B'$ *8*

(b) If $A = \{1,2,3\}$, $B = \{2,3,4\}$, $C = \{1,3,4\}$, $D = \{2,4,5\}$, verify that *8*

$$(A \times B) \cap (C \times D) = (A \cap C) \cap (B \cap D)$$

3. (a) Prove that *8*

$$\begin{vmatrix} x & y & z \\ x^2 & y^2 & z^2 \\ x^3 & y^3 & z^3 \end{vmatrix} = xyz\,(x-z)(y-z)(z-x)$$

(b) If $A = \begin{bmatrix} 2 & 1 & 3 \\ 3 & 1 & 2 \\ 1 & 2 & 3 \end{bmatrix}$, verify that

$$A \cdot (adj \cdot A) = (adj \cdot A) \cdot A = (A)I_3$$ *8*

Unit-II

4. (a) Show that the relation R in the set R of real Numbers defined as $R = \{(a, b) : a \leq b\}$, is reflexive and Transition but not symmetric. 8

(b) If $f(x) = \log \frac{1+x}{1-x}$, show that $f\left(\frac{2x}{1+x^2}\right) = 2f(x)$ 8

5. (a) Evaluate :- 8

(i) $\lim_{x \to 0} \frac{\sqrt{1+x} - \sqrt{1-x}}{2x}$

(ii) $\lim_{x \to 0} \frac{1-\cos 4x}{1-\cos 6x}$

(b) Show that $\lim_{x \to 0} \frac{e^{1/x} - 1}{e^{1/x} + 1}$ does not exist 8

Unit-III

6. (a) If $y = \frac{x}{x+4}$, show that $\frac{xdy}{dx} = y(1-y)$ 8

(b) $\tan^{-1}\left(\frac{\sqrt{1+x^2}+1}{x}\right)$ d.w.r.to x 8

7. (a) If $y = \frac{e^x + e^{-x}}{e^x - e^{-x}}$, find $\frac{dy}{dx}$ 8

(b) If $y=\left(x+\sqrt{x^2+a^2}\right)^4$, then Prove that

$$\frac{dy}{dx}=\frac{xy}{\sqrt{x^2+a^2}}$$ 8

Unit-IV

8. (a) Evaluate 8

(i) $\int\frac{x\sin^{-1}x^2}{\sqrt{1-x^4}}dx$

(ii) $\int e^x\cos x dx$

(b) (i) $\int_1^2\frac{1}{x(1+\log x)}dx$ 8

(ii) $\int_0^{\pi/2}\sqrt{1+\sin x}\ dx$

9. (a) Evaluate 8

(i) $\int\frac{xe^x}{(1+x)^2}dx$

(ii) $\int\frac{\cos x}{(1+\sin x)(2+\sin x)}dx$

(b) (i) $\int_0^4\frac{1}{\sqrt{x^2+2x+3}}dx$

(ii) $\int_0^{\pi/2}\frac{\sqrt{\sin x}}{\sqrt{\cos x}+\sqrt{\sin x}}dx$ 8