

MATHEMATICS

Paper-23BCA401DS01

Mathematical Foundation of Computer Science

Time allowed : 3 hours] [Maximum marks : 70

Note : Students have to attempt **five** questions in total, **first** being compulsory and selecting **one** from each unit. All questions carry equal marks.

1. (a) How many number of subset of a set having  $n$  elements.
- (b) Define equivalence relation ?
- (c) Evaluate  $\lim_{x \rightarrow 0} \frac{\sin x}{x}$
- (d) Discuss the removable discontinuity.
- (e) Find  $\frac{dy}{dx}$  for  $y = \sin^{-1}(x)$ .
- (f) If  $\tan A = \frac{4}{3}$  then evaluate  $\cos A$  and  $\sin A$
- (g) Evaluate the value of  $\int_0^1 \frac{dx}{1+x^2}$ .

Unit-I

2. (a) If  $A = \{2, 4, 6, 8\}$  and  $B = \{6, 8, 10, 12\}$  then find
  - (i)  $A \cup B$
  - (ii)  $A - B$
  - (iii)  $A \cap B$
- (b) In a group of 65-people, 40 like cricket, 10 like both cricket and tennis, find :
  - (i) How many like tennis ?
  - (ii) How many like tennis only and not cricket.

3. (a) If  $f: \mathbb{R} \rightarrow \mathbb{R}$  is defined as  $f(x) = \frac{5x+3}{7}; x \in \mathbb{R}$ . Prove that  $f$  is bijective function and hence find the inverse of  $f$ .
- (b) If the map  $f: \mathbb{R} \rightarrow \mathbb{R}$  is given by  $f(x) = \log(1+x)$  and the map  $g: \mathbb{R} \rightarrow \mathbb{R}$  is given by  $g(x) = e^x$  find  $(g \circ f)x$  and  $(f \circ g)x$ .

Unit-II

4. (a) If  $\tan A = \frac{1}{2}$  and  $\tan B = \frac{1}{3}$ , find  $\tan(2A+B)$ .
- (b) Show that  $\lim_{x \rightarrow 0} \frac{e^{2x}-1}{e^x+1}$  does not exist.

5. (a) Test continuity of the function

$$f(x) = \begin{cases} (x-a) \sin \frac{1}{x-a} & : x \neq a \\ 0 & : x = a \end{cases}$$

- (b) Prove that

$$3 \cos^2 \frac{\pi}{4} + \sec \frac{2\pi}{3} + 5 \tan^2 \frac{\pi}{3} = \frac{29}{2}$$

### Unit-III

6. Differentiate the following function with respect to x.

(a)  $y = \tan^{-1} \left( \frac{\sqrt{1+x^2}-1}{x} \right)$

(b)  $y = \left( \frac{\sin x + e^x}{1 + \log x} \right)$

7. Differentiate the function with respect to x.

(i)  $y = \frac{\sqrt{1+\cos x}}{\sqrt{1-\cos x}}$

(ii)  $y = x^{\log x}$

### Unit-IV

8. (a) Let  $f(x) = x^2 - 5x + 6$ , find  $f(A)$ , if

$$A = \begin{bmatrix} 2 & 0 & 1 \\ 2 & 1 & 3 \\ 1 & -1 & 0 \end{bmatrix}$$

- (b) Prove that  $\begin{vmatrix} x+a & b & c \\ a & x+b & c \\ a & b & x+c \end{vmatrix} = x^3 + (a+b+c)x^2$

9. (a) Solve the given system of linear equations by matrix method.

$$x + y + z = 1, 2x + y + z = 2, x - 2y + z = 4$$

(b) If  $A = \begin{bmatrix} 1 & 3 & 5 \\ -1 & 3 & 7 \\ 0 & -5 & 7 \end{bmatrix}$  and  $B = \begin{bmatrix} 2 & 4 & 6 \\ 0 & -2 & 4 \\ -6 & 8 & 8 \end{bmatrix}$

Prove that  $(AB)^T = B^T A^T$ .